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All-Pay Auctions with Endogenous Bid Timing: An Experimental Study

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Abstract Using a controlled laboratory experiment, we examine individuals' bid timing decisions in complete information all-pay auctions and find that homogeneous bidders are more likely to enter the early bidding stage under a favor-early tie-breaking rule. Furthermore, revenue in an endogenous-entry treatment, in which both sequential and simultaneous all-pay auctions exist, is either lower than or equal to that in an exogenous-entry treatment with only simultaneous all-pay auctions. Additionally, in simultaneous all-pay auctions, individuals do not always employ a mixed strategy as predicted by the risk-neutral model. Instead, our data is better rationalized by a risk- and loss-aversion model.

Keywords: all-pay auction, bid timing, experiment, mixed-strategy Nash equilibrium
JEL Classification: C7, C91

I would like to thank Yan Chen, Lada Adamic, Liuling Gong, Audrey Hu, Erin Krupka, Zheng Li, Baoping Liu, Jeffrey Mackie-Mason, Yusufcan Masatlioglu, Scott Page, Kan Takeuchi, Neslihan Uler, and the seminar participants at the North American Regional Meetings of the ESA (Tucson, AZ, November 2010) and the 2011 International Meetings of the ESA (Chicago, IL, July 2011). I thanks Robert Ketcham for his excellent research assistance in conducting the lab experiments. I am grateful to two anonymous referees and the Associate Editor for helpful comments and suggestions, which significantly improved the paper. The financial support from the School of Information and the Rackham Graduate School, University of Michigan, Tsinghua University, the National Natural Science Foundation of China (NSFC) under grant 71403140, is greatly appreciated.

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1 Introduction

All-pay auctions have been used to model diverse activities such as political lobbying, research and development (R&D) races, and tournaments. Recently, these auctions have been used as a major exchange mechanism in emerging online labor markets, known as crowdsourcing contests. In these online labor markets, a task is posted by a requester along with a reward for the best solution, and any user of the website may submit a solution. The requester subsequently selects the best solution and rewards the corresponding user, while other users' efforts are not compensated for. If we equate users' submission quality with bids in an auction, the system is approximately equivalent to an all-pay auction, especially for an expertise-based task where a contestant's performance is mainly determined by ability and effort (Terwiesch and Xu 2008).

Most relevant literature concerns simultaneous all-pay auctions in which players bid simultaneously and independently. However, in many real-life applications, individuals or groups move sequentially instead of simultaneously, and later players can decide how much effort to expend after observing early players' behavior. Examples of sequential contests include patent competition (Leininger 1991), and the United States presidential election process (Morgan 2003). Moreover, the choice between sequential and simultaneous all-pay auctions is an open call for contest designers (e.g., crowdsourcing contests), and which mechanism is more effective for encouraging better performance is currently subject to experimentation. Many websites such as Taskcn.com, implement sequential all-pay auctions where users submit their solutions sequentially and late entrants observe the contents of prior solutions. Conversely, websites such as Topcoder.com use simultaneous all-pay auctions that prevent users from reading other submissions. Sites such as 99design.com allow contest holders to choose between these two mechanisms.

Moreover, participants in real-life situations endogenously decide when to enter a contest, and the bid timing decisions determine the format of the contest. For example, the contest becomes simultaneous if all contestants on Taskcn.com choose to password-protect their solutions, and sequential otherwise. Furthermore, a tie-breaking rule, which favors early submissions, is often used in practice. Many requesters on crowdsourcing contest sites explicitly indicate that, in the case of multiple best solutions, the earliest submission will be selected.[1] The incumbent in a market also has an advantage against the new entrant because of consumers' stickiness (Giulietti, Price and Waterson 2005, Schlesinger and Von der Schulenburg 1991, Waterson 2003). Therefore, we investigate whether this simple tie-breaking rule could effectively affect individuals' bid timing decisions.

In order to do this, we examine individuals' bid timing decisions in all-pay auctions, in which players first endogenously choose which bidding stage to enter, and we manipulate the use of different tie-breaking rules. To characterize different competition environments, we vary bidding costs so that players have either identical or different bidding costs, and compare revenue in all-pay auctions with endogenous against exogenous bid timing, i.e., exogenous simultaneous all-pay auctions. We find

[1] An example is http://www.taskcn.com/w-60017.html, retrieved on October 15, 2011.

that a tie-breaking rule that favors early bidders significantly induce earlier bids for homogeneous players but not for heterogeneous ones. The revenue in an endogenous-entry treatment is either equal to or less than that in an exogenous-entry treatment. In addition, individuals in simultaneous all-pay auctions do not always bid as predicted by a risk-neutral model, and a risk- and loss-averse model can better rationalize the data.

The rest of this paper is organized as follows. In Section 2, we review the relevant literature. Section 3 presents a theoretical model of all-pay auctions with endogenous bid timing. Section 4 presents the experimental design. In Section 5, we present experimental results. Section 6 concludes the paper.

2 Literature Review

In all-pay auction literature, most studies explore simultaneous all-pay auctions.[2] For complete information games in which each bidder's value (or bidding cost) is common knowledge, Baye, Kovenock and de Vries (1996) provide a complete characterization of the mixed strategy Nash equilibria. Particularly, total bids are expected to be less than or equal to the value of the object. Using the theory prediction in Baye et al. (1996), a series of experiments (Davis and Reilly 1998, Gneezy and Smorodinsky 2006, Lugovskyy, Puzzello and Tucker 2010) report overbidding where total bids exceed the value of the object. This phenomenon is attributed to various factors, including (1) the size of the group; (2) individual experiences; and (3) the matching protocol. In an experimental study with a minimum group size (2 players), a sufficient learning opportunity (30 rounds), and a random rematching protocol, Potters, de Vries and van Winden (1998) find that the average bid is consistent with the NE prediction. Consistently, no significant over (under) bidding is observed in Grosskopf, Rentschler and Sarin (2010) which has similar experimental protocol. Moreover, Potters et al. (1998) report that bidders can be categorized into different types. For example, a substantial proportion choose to overweigh strategies with higher realized payoffs in earlier rounds; consequently, they are more likely to use the same strategy across rounds instead of randomizing their bids. Additionally, both Klose and Sheremeta (2012) and Ernst and Thöni (2013) find that the bids exhibit a bimodal distribution instead of a uniform one, and the reference dependent model, which incorporates both risk and loss preference, can organize the data.

Numerous studies examine simultaneous all-pay auctions with incomplete information. Assuming an i.i.d. distribution of bidder type and risk neutrality, the unique monotonic symmetric Bayesian NE has been obtained in prior studies (Chawla, Hartline and Sivan 2012, Hillman and Riley 1989, Krishna and Morgan 1997, Weber 1985). A few theoretical works consider independent types drawn from different distributions (Amann and Leininger 1996, Kirkegaard 2012). Other studies examine interdependent types (Krishna and Morgan 1997, Rentschler and Turocy 2016, Siegel 2014). For example, under the monotonicity condition, Siegel (2014) analyzes

[2] There is extensive literature on lottery contests in which the winning probability is not deterministic but proportional to the number of bids (Tullock 1980). We refer the reader to Dechenaux, Kovenock and Sheremeta (2015) for a summary of this literature and the references therein.

(a)symmetric two-player all-pay auction with finite sets of types. Rentschler and Turocy (2016) further relax the monotonicity assumption, and characterize all symmetric equilibria. Olszewski and Siegel (2015) apply the mechanisms design approach to approximate equilibrium behavior and to solve efficient prize allocation in large contests.

Regarding experimental research, overbidding has also been observed and found to decrease with experience (Noussair and Silver 2006). Players with a low value often bid lower than the equilibrium prediction whereas high-value players tend to bid higher than the equilibrium prediction (Muller and Schotter 2010, Noussair and Silver 2006), and this can be explained by risk-aversion (Fibich, Gavious and Sela 2006) and (or) loss-aversion (Mermer 2013).

Relative to the extensive literature on simultaneous all-pay auctions, there are few studies on sequential all-pay auctions. Konrad and Leininger (2007) characterize the subgame perfect NE in a complete information all-pay auction with endogenous bid timing. Segev and Sela (2014) derive the perfect Bayesian NE for an exogenously sequential all-pay auction with incomplete information. Following Segev and Sela (2014), Jian, Li and Liu (2016) characterize the explicit expression for the expected highest bids for sequential all-pay auctions with n-players and show that it is lower than that in simultaneous all-pay auctions. Additionally, they use a laboratory experiment to confirm this theoretical prediction.

Furthermore, a growing stream of experiments examines endogenous entry in contests. Some of them examine the entry choice between rank-order tournaments and other incentive structures, e.g., piece-rate (Bartling, Fehr, Marechal and Schunk 2009, Dohmen and Falk 2011, Eriksson, Teyssier and Villeval 2009) and fixed payment (Anderson and Stafford 2003, Dohmen and Falk 2011, Morgan, Orzen and Sefton 2012). Specifically, when individuals are less risk-averse, they are more likely to choose tournaments over other payment schemes (Dohmen and Falk 2011, Eriksson et al. 2009). Moreover, high-cost players are less likely to enter contests and prefer fixed-payments (Anderson and Stafford 2003). Other studies focus on entry choice between different types of contests, such as single v. multiple prize contests (Vandegrift and Yavas 2010), and single v. proportional prize contests (Cason, Masters and Sheremeta 2010). Particularly, Cason et al. (2010) find that, compared to single-prize contests, proportional-prize contests induce more entries and generate higher total effort by encouraging the performance of low-ability contestants.

Compared to the aforementioned studies which examine the entry choice between different payment schemes, our focus is on bid timing decisions, i.e., bidding at an earlier or later stage. In other words, we assume that individuals voluntarily participate in a contest. However, before the contest starts, they need to decide when to bid, and their bid timing decision jointly determines the contest structure, i.e., sequential v. simultaneous contests. Altogether, this study provides the first experimental examination of individuals' bid timing decisions in endogenous all-pay auctions. Furthermore, we examine the impact of tie-breaking rules on bid timing decisions. Next, we compare the performance of all-pay auctions with endogenous bid timing and those with exogenous bid timing, i.e., an exogenously simultaneous all-pay auction. Last but not least, we examine individual strategy in simultaneous all-pay auctions with complete information, i.e., whether players bid uniformly on their bid space.

3 Theoretical Framework

In an all-pay auction, an object with value v is auctioned between two bidders.[3] Bidder i has a linear cost function, $c_i(x_i) = c_i x_i$, where x_i is her bid. $\bar{x}_i$ is the reach, determined by $c_i \bar{x}_i = v$. Without loss of generality, we assume that $c_1 \leq c_2$. For a risk-neutral bidder, her payoff, $\pi_i(x_i, x_j)$, is:

$$\pi_i(x_i, x_{-i}) = \begin{cases} v - c_i x_i & x_i > x_j, \\ -c_i x_i & \text{otherwise} \end{cases}$$

The equilibrium for simultaneous all-pay auctions is characterized in Baye et al. (1996) and summarized below:

1. When $c_1 = c_2 = c$, the unique NE is that both players randomize uniformly on [0, $\frac{v}{c}$], and the expected revenue is $\frac{v}{c}$.
2. When $c_1 < c_2$, the unique NE is that player 1 randomizes uniformly on [0, $\bar{x}_2$]. Player 2 randomizes uniformly on (0, $\bar{x}_2$] and her probability of bidding 0 is $1 - c_1$. Additionally, the expected revenue is $\frac{v}{2c_2} + \frac{vc_1}{2c_2^2}$.

In a sequential all-pay auction, the late bidder submits her bid after observing the early bid. Following Konrad and Leininger (2007), we assume a favor-late tie-breaking (LTB) rule in our theoretical analysis to derive the strict SPNE. Specifically, when there is a tie in the sequential game, the late bidder is always the winner.

When other tie breaking rules are used, e.g., the random tie-breaking rule and the favor-early tie-breaking rule (ETB), we are not able to pin down the best response function for bidders and the strict SPNE does not exist.[4] For example, for homogeneous bidders, if the early bid $x_e < v/c$, there is no optimal bid for the late bidder under the continuous bid space. Additionally, a set of ϵ equilibria exists and lies in ϵ-neighborhood of the strict SPNE (Konrad and Leininger 2007). If we assume that people do not distinguish between infinitesimally small amounts of money such as 0.01 cent from zero and follow the argument of ϵ equilibria, we should not expect significant behavioral differences under different tie-breaking rules.

When $c_1 = c_2 = c$, the SPNE is characterized below:

1. With the early bid: x_e, the best response function for the late bidder is:

$$x_l^* = \begin{cases} x_e & x_e < v/c, \\ \{0, v/c\} & x_e = v/c, \\ 0 & \text{otherwise.} \end{cases}$$

2. There are two equilibrium strategies for the early bidder: $x_e^* = 0$ and $x_e^* = v/c$.

When $c_1 < c_2$, the SPNE is analyzed in Konrad and Leininger (2007) and we reproduce it here.

[3] We relegate the analysis for n players to the appendix.

[4] We would like to thank one of the anonymous referees for pointing this out.

1. The best response function for the late bidder is identical to the homogeneous case.

$$x_l^* = \begin{cases} x_e & x_e < v/c_l, \\ \{0, v/c_l\} & x_e = v/c_l, \\ 0 & \text{otherwise.} \end{cases}$$

2. When the early bidder is the low-cost player, the equilibrium strategy is: $x_e^* = v/c_2$.[5] When it is the high-cost player, the equilibrium strategy is: $x_e^* = 0$.

Next we extend the model to an endogenously bid timing case. Before the auction starts, the two players independently and simultaneously decide between the early and late bidding stages. Everyone's bid timing decision is then publicly announced, and their bid timing decisions jointly determine the structure of the subsequent all-pay auctions. The probability of entering the early bidding stage is q_i for bidder i.

Assuming players follow the equilibrium play in the subgame, we analyze the optimal bid timing decision for each player. For homogeneous bidders, conditional on the subgame where the equilibrium outcome is that both early and late players bid 0, the equilibrium payoff for the early bidder is 0 and it is v/c for the late bidder. Therefore, $q_i^* = 0$ is the best response if $q_j > 0$; otherwise, $q_i^* \in [0, 1]$. In contrast, conditional on the subgame where the equilibrium outcome is that the early player bids v/c and the late player bids 0, the equilibrium payoff for both players is 0. Consequently, $q_i^* \in [0, 1]$.

For heterogeneous bidders, if $q_2 > 0$, the expected payoff for the low-cost player 1 to enter late is always higher than entering early. Therefore, $q_1^* = 0$. However, if $q_2 = 0$, her expected payoff between entering early and late is always the same. Therefore, $q_1^* \in [0, 1]$. Additionally, since the expected payoff for the high-cost player 2 is always 0, $q_2^* \in [0, 1]$.

4 Experimental Design

First, we identify two bidding cost environments. In the homogeneous environment, the marginal bidding cost is 1 token/bid for both bidders. In the heterogeneous environment, one player is randomly selected as the high-cost bidder at the beginning of each round and her bidding cost is 1 token/bid, while the low-cost bidder pays 0.8 token/bid. Second, we implement both endogenous- and exogenous entry treatments, and the latter is used to replicate results from the relevant literature (Davis and Reilly 1998, Gneezy and Smorodinsky 2006, Lugovskyy et al. 2010). In endogenous-entry treatments, we include both ETB and LTB to examine the effect of tie breaking rules, yielding both endogenous-entry-ETB and endogenous-entry-LTB treatments.

Altogether, we have a 2×3 factorial design (Table 1). Each treatment has three independent sessions, with 12 subjects in each session. At the beginning of each round, subjects are randomly matched into groups of two. Since our model assumes one-shot interactions, the random re-matching protocol minimizes repeated-game effects. The value of the object is always set at 100 tokens, and the entire structure of the game

[5] Strictly speaking, it is $x_e^* = v/c_2 + \epsilon$, and when $\epsilon \to 0$, $x_e^* \to v/c_2$.

Table 1: Experimental Design

Auction Format	Bid Timing	Tie-Breaking Rule	Homogeneous Bidders	Heterogeneous Bidders	Total Subjects
Sim	Exogenous	Random	12×3	12×3	72
Sim or Seq	Endogenous	ETB	12×3	12×3	72
Sim or Seq	Endogenous	LTB	12×3	12×3	72

is common knowledge. To prevent any potential bankruptcy problems, we give every subject 125 tokens as an endowment at the beginning of each round. Finally, each session lasts 30 rounds to capture any learning effect.

In exogenous-entry treatments, there is no entry decision stage and every subject submits a bid independently and simultaneously. At the end of each round, the highest bidder wins and each bidder pays for her own bid. If there is a tie for the highest bid, the winner is chosen randomly.

Conversely, in endogenous-entry treatments, each player first chooses to enter the early vs. late bidding stage. After the entry decisions are shared between the two players, each of them chooses a bid in her respective bidding stage. When both players choose the same stage, they bid independently and simultaneously and the tie is randomly broken, as in exogenous-entry treatments. When they enter different bidding stages and submit the same bid, the early (late) bidder is the winner in the ETB (LTB) treatment. In other words, the tie-breaking rule difference between ETB and LTB treatments only occurs when a sequential all-pay auction is formed. A sample of the instructions is included in Appendix B.

After players participate in 30 rounds, we implement the lottery choices outlined in Tanaka, Camerer and Nguyen (2010) to measure individual risk preference. Then we give each participant a post-experiment questionnaire which includes demographic and personality trait questions.[6]

Altogether, we conducted 18 independent computerized sessions at the School of Information Lab at the University of Michigan in May 2010, utilizing a total of 216 subjects. Subjects were students at the University of Michigan, recruited by email from a subject pool for economic experiments. We allowed subjects to participate in only one session. We used z-Tree (Fischbacher 2007) to program our experiments. Each session lasted approximately one hour, with the first 15 minutes used for instructions. The exchange rate was set at 8 tokens per USD. Additionally, each participant was paid a $5 show-up fee. The average amount participants earned was $20, including the show-up fee. Data are available from the author upon request.

[6] Both the lottery choices and the post-experiment survey are included in online appendices.

5 Results

In this section, we first report entry decisions in endogenous-entry treatments and compare performance between endogenous- and exogenous-entry treatments. Subsequently, we investigate whether individuals randomize in simultaneous all-pay auctions.

Three common features apply throughout our analysis. First, for non-parametric tests, we treat each session as one independent observation and compute the average within the session across multiple rematched pairs and all periods. Therefore, we have three independent observations per treatment. Second, for regression analyses, standard errors are also clustered at session level, which allows any form of correlations among observations within a session. Finally, two-sided p-values are reported.

5.1 Entry Decisions

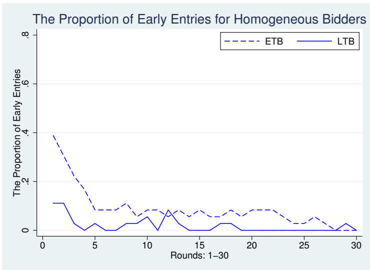

Figure 1: The Proportion of Entering Early for Homogeneous Bidders

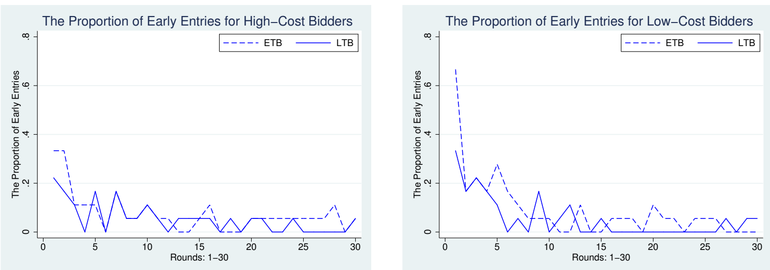

Figure 2: The Proportion of Entering Early for Heterogeneous Bidders

First, Konrad and Leininger (2007) predict that compared to LTB, other tie-breaking rules, e.g., ETB, would produce ϵ equilibria that lie in ϵ- neighborhood of the strict SPNE. Therefore, by following the argument of ϵ equilibria, we have the following hypothesis.

Hypothesis 1 (Effects of Tie-Breaking Rules) *In endogenous-entry treatments, tie-breaking rules do not affect bid timing decisions.*

Figure 1 presents the proportion of early entries for homogeneous bidders under different tie-breaking rules. The dashed line refers to the ETB treatment, while the solid line is the LTB treatment. Although the proportion of early entries decreases with time, it is always higher in ETB than LTB, except in rounds 12 and 29. In contrast, no significant difference exists for heterogeneous bidders (Figure 2). By using a test of proportions with standard errors clustered at session level, we summarize the results below.

Result 1 (Entry Decisions) *Homogeneous bidders are significantly more likely to enter early under ETB than LTB (9% v. 2%, $p = 0.005$), while there is no significant difference for heterogeneous bidders (low-cost : 9% v. 5%, $p = 0.407$; high-cost: 8% v. 5%, $p = 0.615$).*

In contrast with Hypothesis 1, Result 1 suggests that ETB induces significantly more early bids for homogeneous bidders, but not for heterogeneous bidders. One possibility is that, homogeneous bidders may expect that the early bidder garners more advantage to win the game when ETB is implemented. However, as the winning probability for the low-cost player is obviously higher than the high-cost one, the tie-breaking rule may not affect their bid timing decisions.[7]

Although the theory does not have a clear prediction for the likelihood of entering into the late stage for the high-cost player, i.e., q_2^* can be any number in [0,1], our experiment reveals that these players overwhelmingly choose the late bidding stage. This extends the use of experimental methods to solve the equilibrium selection problem in games with multiple Nash equilibria (Brandts and Holt 1993, Cabrales, Nagel and Armenter 2007, Chen and Chen 2011, Goeree and Holt 2005).

We conjecture that conditional on low-cost players choosing late, high-cost players may still have more incentive to enter late and participate in a simultaneous game rather than staying early and getting zero payoff for sure. Though the expected payoff in simultaneous all-pay auction is always zero,[8] their winning probability is positive and expost payoff can be positive. For example, the theory predicts that high-cost bidders have a 40% chance of winning in simultaneous all-pay auctions, and, in the experiment, it is 34% for ETB and 35% for LTB. Additionally, since a certain proportion of low-cost bidders enter early and bid less than 100 (Figure 5 in Section 5.2), entering late is a best reply for high-cost bidders.

5.2 Endogenous v. Exogenous All-Pay Auctions

In this section, we examine individual bids in both sequential and simultaneous all-pay auctions. Moreover, we compare revenue, players' earnings and efficiency be-

7 We would like to thank one of the anonymous referees for suggesting this explanation.

8 In Section 5.3, we show that the expected payoff for high-cost players is always zero even under the risk- and loss-averse assumption (Proposition 1).

tween treatments. Assuming the existence of two types of sequential games, especially the one in which the early player bids 0, we have the following hypothesis.

Hypothesis 2 (Endogenous v. Exogenous All-Pay Auctions: Homogeneous Bidders) *For homogeneous bidders, the revenue in the endogenous-entry treatment will be less than that in the exogenous-entry treatment.*

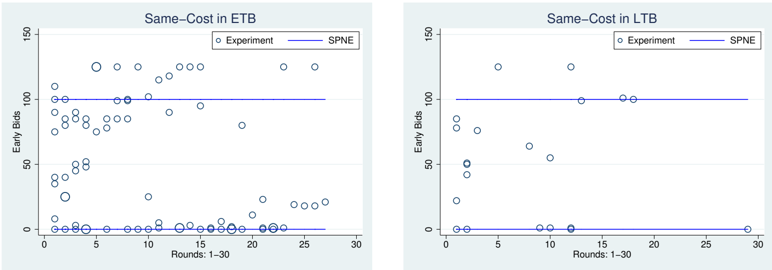

Figure 3: Early Homogeneous Bidders in Sequential All-Pay Auctions

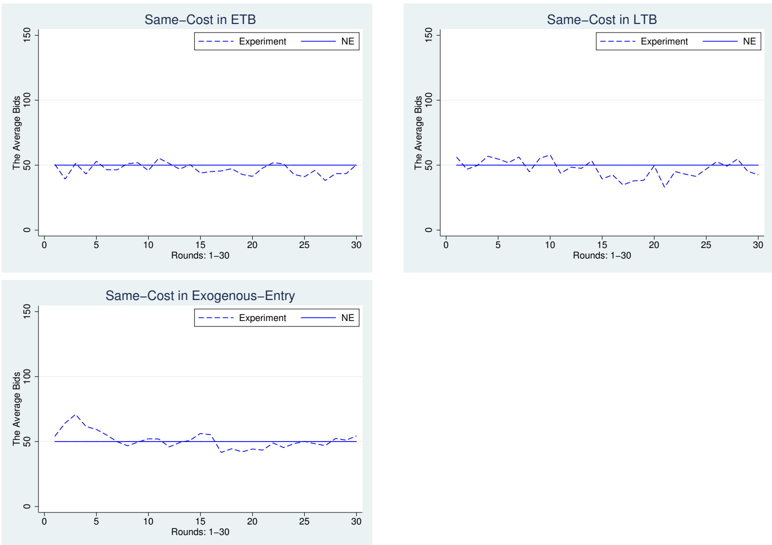

Figure 4: Homogeneous Bidders in Simultaneous All-Pay Auctions

For homogeneous bidders in sequential all-pay auctions, Figure 3 presents the early bids with ETB (left panel) and with LTB (right panel), respectively. The circles represent individual bids and the size of the circle indicates the number of observations for the particular bid. The solid lines are bids predicted by SPNE. With ETB, many conform to equilibria characterization, i.e., 20% of them are equal to or above 100, while 36% are equal to or near 0.[9] However, as our SPNE predictions are at the boundary (Kimbrough, Sheremeta and Shields 2014, Laury and Holt 2008, List 2007), a substantial proportion of bids still deviates from SPNE.[10] Additionally, 80% of late bidders under ETB best respond to their early bidders. Similarly, early bids are also bifurcated under LTB, while we are not able to draw any robust conclusions because of limited number of observations.

In simultaneous all-pay auctions, Figures 4 presents the average bid in endogenous- and exogenous-entry treatments, respectively.[11] Overbidding is observed in early rounds, but decreases in later rounds. Aggregately there is no significant difference from NE prediction ($p > 0.1$, signed-rank tests). Furthermore, the pairwise comparison between treatments is not significant either ($p > 0.1$, rank-sum tests), indicating that whether individuals endogenously choose or are exogenously assigned to simultaneous all-pay auctions does not affect their bids.

Table 2: Determinants of Revenue in All-Pay Auctions

Dependent Variable	Homogeneous (1)	Heterogeneous (2)
ETB	-10.10	-13.40**
	(8.31)	(4.30)
LTB	-8.20	5.66
	(9.91)	(4.22)
Round	-0.62***	-1.71***
	(0.14)	(0.35)
Constant	112.00***	136.40***
	(7.81)	(4.66)
Observations	1,613	1,611
R^2	0.017	0.070

Notes: 1. Standard errors are in parentheses.
2. Significant at: * 10%; ** 5%; *** 1%.

Finally, we report the revenue comparison between treatments. Table 2 presents OLS regression results. The dependent variable is the revenue in each auction, the independent variables are ETB, LTB treatment dummies, and we control for learning effect by using the round variable. As a certain proportion of early players bid 100 or above in sequential all-pay auctions, the treatment dummies are not significant for homogeneous bidders (column 1 in Table 2). In addition, the round variable is

9 Following Gneezy and Smorodinsky (2006), we use bids ≤ 5 as the cutoff for “near-zero” bids. Moreover, the results are qualitatively similar when different cutoffs are used.

10 We would like to thank one of the anonymous referees for pointing this out.

11 Because only 1% of simultaneous all-pay auctions in endogenous treatments have both bidders in the early stage, we focus on simultaneous all-pay auctions with two late bidders.

negative and significant at the 1% level, suggesting a salient learning effect on the amount of revenue.[12]

Result 2 (Revenue: Homogeneous Bidders) *For homogeneous bidders, the revenue in the endogenous-entry treatment is not significantly different from that in the exogenous-entry treatment.*

By Result 2, we can not reject the null in favor of Hypothesis 2. Consistently, average earnings are not significantly different between treatments ($p > 0.1$, rank-sum tests).

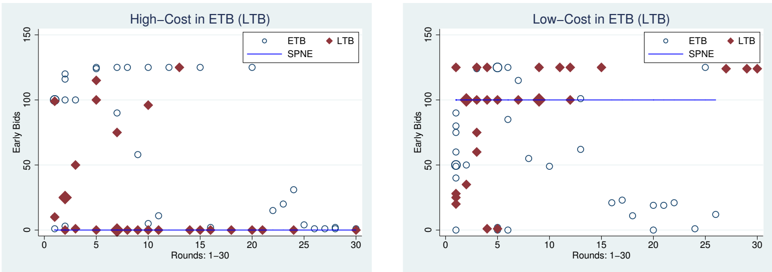

Figure 5: Early Heterogeneous Bidders in Sequential All-Pay Auctions

Subsequently, we examine revenue for heterogeneous bidders. Assuming the existence of a sequential game in which the early high-cost player bids 0, we have the following hypothesis.

Hypothesis 3 (Endogenous v. Exogenous All-Pay Auctions: Heterogeneous Bidders) *For heterogeneous bidders, the revenue in the endogenous-entry treatment will be less than that in the exogenous-entry treatment.*

Figure 5 plots early bids in sequential all-pay auctions. The circles (diamonds) represent ETB (LTB), and the solid line is SPNE. For high-cost bidders, their bids are bifurcated at the beginning and decrease significantly with experience. After round 15, the proportion of "near-zero" bids is 79%, and moves to 100% in the last five rounds. For low-cost ones, their bids are bifurcated even in the last five rounds.

Figure 6 presents the average bid for both high- and low-cost bidders in simultaneous all-pay auctions. Consistent with homogeneous bidders, both players overbid at the beginning and decrease their bids with experience. Moreover, the average bid for both bidders is not significantly different from NE ($p > 0.1$, signed-rank tests). There is no significant treatment difference for low-cost bidders. Surprisingly, the average bid for high-cost bidders in the ETB treatment is significantly lower than the other two treatments (ETB v. LTB: 36 v. 48, $p = 0.050$; ETB v. Exogenous: 36 v. 49, $p = 0.050$; LTB v. Exogenous: 48 v. 49, $p = 0.513$, rank-sum tests).

[12] The pairwise comparisons from non-parametric tests are consistent with regression results (ETB v. LTB: 92 v. 94, $p = 0.513$; ETB v. Exogenous: 92 v. 102, $p = 0.513$; LTB v. Exogenous: 94 v. 102, $p = 0.827$, rank-sum tests).

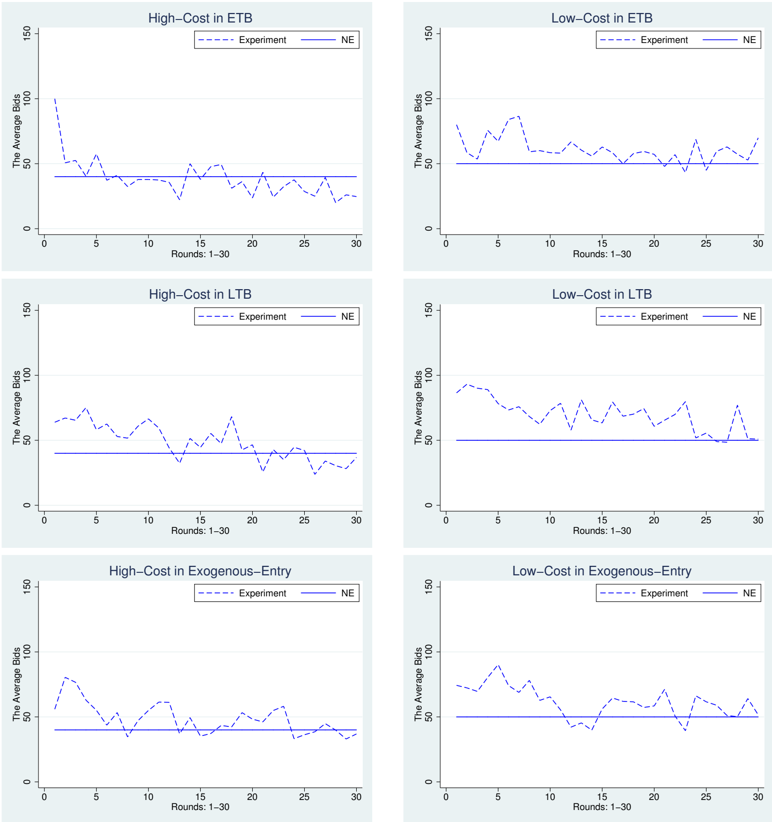

Figure 6: Heterogeneous Bidders in Simultaneous All-Pay Auctions

Finally, we compare the revenue for heterogeneous bidders between treatments. Column 2 in Table 2 reports regression results. As in the ETB treatment, 50% of high-cost early bidders choose 0 and their bids in simultaneous all-pay auctions are also lower than in other treatments, the estimated coefficient of ETB is negative and significant at the 5% level. However, there is no significant difference between LTB and exogenous-entry treatments.[13] We summarize the results below:

Result 3 (Revenue: Heterogeneous Bidders) *For heterogeneous bidders, the revenue in the ETB treatment is significantly lower than the exogenous-entry treatment.*

Result 3 rejects the null in favor of Hypothesis 3 for the ETB treatment. Additionally, the average earnings in each round are higher in the ETB treatment than in the other two treatments (ETB v. LTB: 8 v. -1, $p = 0.050$; ETB v. Exogenous: 8 v. 1, $p = 0.050$, rank-sum tests).

[13] The non-parametric tests present the same results (ETB v. LTB: 96 v. 115, $p = 0.05$; ETB v. Exogenous: 96 vs. 110, $p = 0.05$; LTB v. Exogenous: 115 vs. 110, $p = 0.513$, rank-sum tests).

Table 3: Efficiency Comparison for Heterogeneous Bidders

Treatment	Efficient Allocation	Utilization Ratio
ETB	64%	1.34
LTB	66%	1.40
Exogenous	59%	1.38
ETB v. LTB	0.827	0.050
ETB v. Exogenous	0.513	0.127
LTB v. Exogenous	0.127	0.513

Note: p-values are computed using rank-sum tests.

Additionally, we discuss the efficiency for heterogeneous bidders. Based on prior studies (Plott and Smith 1978, Noussair and Silver 2006), an efficient allocation is that the winner is the low-cost bidder. Furthermore, since everyone's effort is forfeited in an all-pay auction, another way to evaluate efficiency is to measure the amount of wasted effort. The lower the wasted effort, the higher the efficiency. Chawla et al. (2012) define a "utilization ratio" to quantify this efficiency measurement. It is defined as the ratio of expected total bids by all participants to the expected highest bids.[14] Table 3 presents summary statistics for both efficiency measurements. First, the proportion of efficient allocation in the endogenous-entry treatment is higher than in the exogenous-entry treatment, although the comparison is not statistically significant. Moreover, the utilization ratio in the ETB treatment is lower than in the other two treatments. Particularly, the comparison between ETB and LTB is significant at the 5% level ($p = 0.050$). This suggests that, although the ETB treatment generates lower revenue compared to others, it has the advantage of lowering the amount of waste in all-pay auctions.

5.3 Individual Strategy Analysis in Simultaneous All-Pay Auctions

In this section, we investigate whether bidders, whose average bid is consistent with NE predictions in simultaneous all-pay auctions, have the same bidding distribution as shown by NE predictions with risk-neutral bidders. It is worth noting that bids in experiments are discrete, and a continuum of (a)symmetric NE exists (Baye, Kovenock and de Vries 1994, Bouckaert, Degryse and Vries 1992, Li 2015). However, as both the reward size and the bid space in our experiment are relatively large, we use the NE prediction under the continuous bid space as the basis for hypotheses testing.

Hypothesis 4 (Individual Strategy in Simultaneous All-pay Auctions) *In simultaneous all-pay auctions, homogeneous and low-cost bidders randomize uniformly on [0, 100]. The high-cost bidder randomizes uniformly on (0, 100] and the probability for bidding 0 is 0.2.*

[14] We compute the utilization ratio at the session level where the expected total bids are approximated by the average revenue and the expected highest bid is the average highest bid.

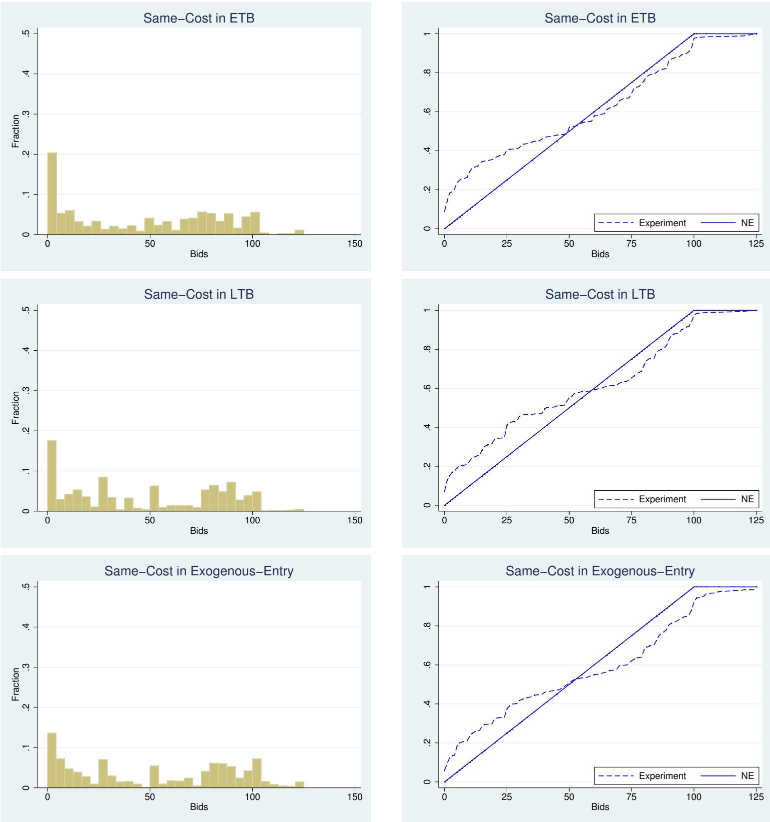

Figure 7: Homogeneous Bidders in Simultaneous All-Pay Auctions

Table 4: Percentage of Each Bidder Type in Simultaneous All-Pay Auctions

Bidder Type	Homogeneous	Low-Cost	High-Cost	All
Under-Bidders	14	11	8	11
Over-Bidders	13	4	7	8
Random-Bidders	20	47	53	40
Others	53	38	31	41

Figures 7, 8 and 9 present the respective bidding histograms for each type of bidders (left column), the bidding CDF for both actual bids and NE predictions (right column). In contrast with NE, the bidding distribution largely follows a bimodal pattern (Ernst and Thöni 2013, Klose and Sheremeta 2012). Furthermore, the proportion of near-zero bids is also higher than NE for high-cost bidders, e.g., it is 42% under ETB. Additionally, low-cost bidders often choose to bid 125, suggesting the importance of the “joy of winning” (Parco, Rapoport and Amaldoss 2005, Sheremeta 2010).

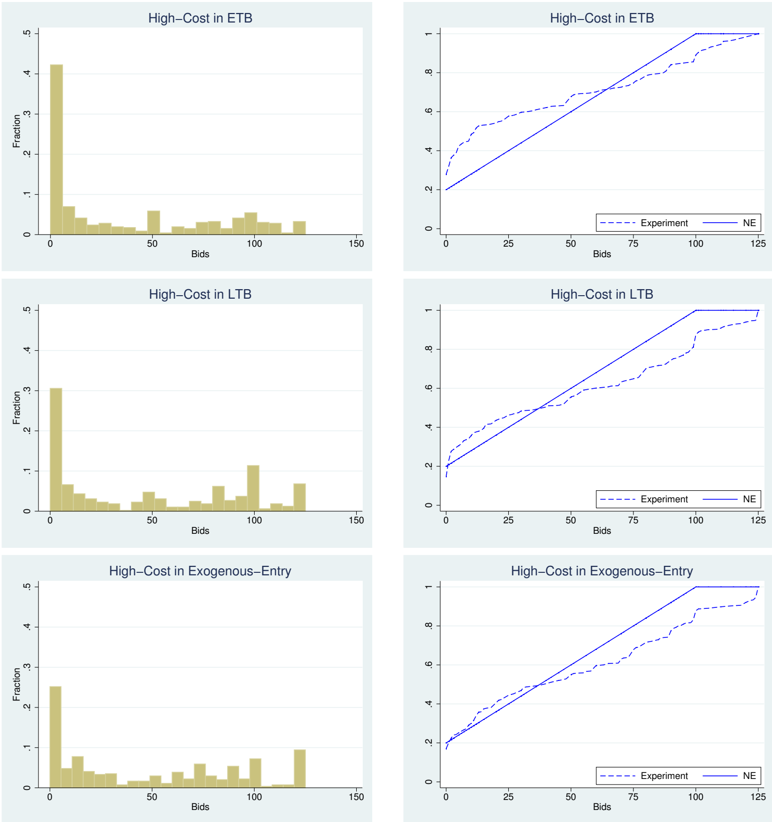

Figure 8: High-Cost Bidders in Simultaneous All-Pay Auctions

Applying the econometric model in Wooders (2010) and Walker and Wooders (2001), we further confirm that bids are not uniformly distributed in [0, 100]. Using the expected bid under the risk-neutral model as the cutoff, which is defined as c_0, we categorize players into four types: (1) under-bidders: those who consistently bid less than c_0; (2) over-bidders: those who consistently bid more than c_0; (3) random-bidders: those who are equally likely to bid between $[0, c_0]$ and $(c_0, 100]$; and (4) others: those who neither randomize nor maintain the same strategy. Table 4 lists the percentage for each type. We summarize this result below and the details of the analysis are described in online appendices.

Result 4 (Individual Bids in Simultaneous All-pay Auctions) *In simultaneous all-pay auctions, 40% of bidders randomize their bids, 8% overbid, and 11% underbid.*

In contrast with Hypothesis 4, Result 4 indicates that bidding strategies are heterogeneous in simultaneous all-pay auctions and individuals do not necessarily play uniformly. Following the all-pay auction literature, we extend the model to include risk- and loss-aversion (Fibich et al. 2006, Noussair and Silver 2006, Ernst and

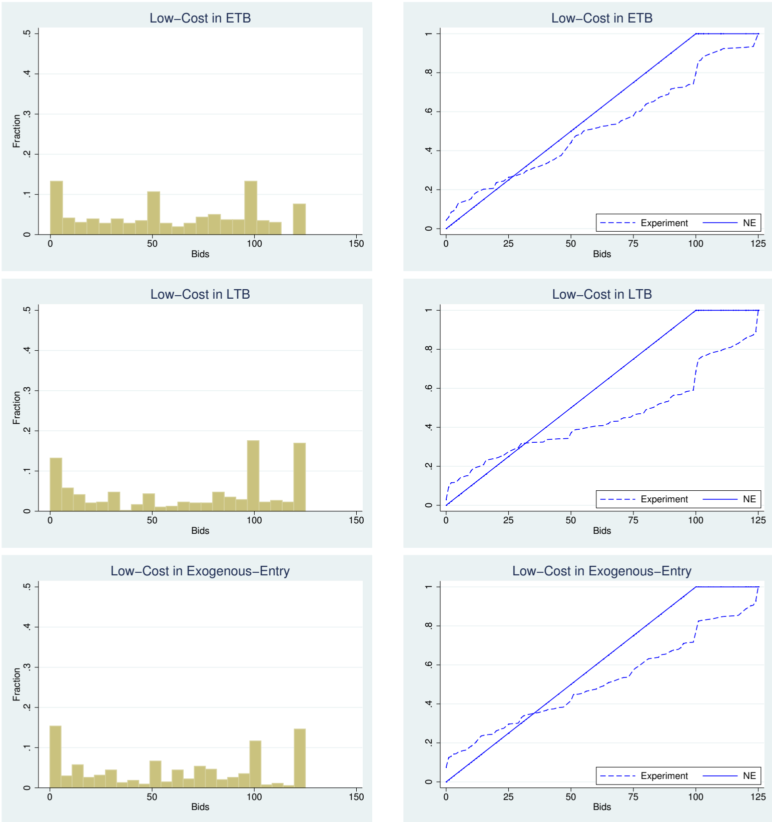

Figure 9: Low-Cost Bidders in Simultaneous All-Pay Auctions

Thöni 2013, Klose and Sheremeta 2012). Assuming α is the risk-aversion parameter, where $0 < \alpha \leq 1$, λ is the loss-aversion parameter with $\lambda \geq 1$, and normalizing $c = c_2 = 1$, we obtain the following proposition for n players.

Proposition 1 (Simultaneous All-Pay Auctions with Risk- and Loss-Averse Bidders)

1. *The unique symmetric NE is still the mixed strategy NE where individuals randomize on [0, v].*[15]
2. *For all active bidders in the equilibrium, the bidding CDF:* $G_i(x)$*, satisfies:* $G_i''(x) < 0$ *with* $0 \leq x \ll v$ *and* $G_i''(x) > 0$ *with* $0 \leq v - x \ll v$*.*
3. *For bidder* 2 *among heterogeneous bidders,* $G_2(0) \geq G_2^{rn}(0)$*.*[16]

[15] For homogeneous bidders, all players randomize, while for heterogeneous bidders, only players 1 and 2 randomize and others always bid 0.

[16] $G_2(0)$ is bidder 2's probability of bidding 0. $G_2^{rn}(0)$ is her probability of bidding 0 with risk and loss neutrality.

Proof See Appendix A.

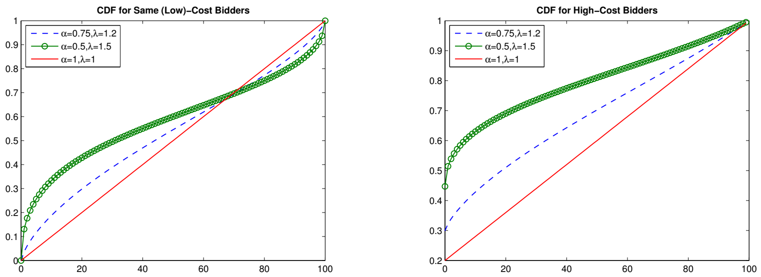

Figure 10: Bidding CDF for Risk- and Loss-Averse Bidders in Simultaneous All-Pay Auctions

Figure 10 presents two numerical examples. In each graph, the solid line represents the risk-neutral model and the dots (dashed line) represent relatively higher (lower) risk- and loss-aversion. The bidding CDF exhibits an inverse S-shape; in particular, high-cost bidders have a higher probability of bidding 0 than those in the risk-neutral model. Additionally, higher risk-aversion implies more bifurcated individual bids, and higher loss-aversion implies a greater number of lower bids.

Subsequently, similar to Ernst and Thöni (2013) and Klose and Sheremeta (2012), we estimate risk- and loss-aversion parameters using our experimental data. Table 5 reports the Maximum Likelihood Estimation results. Both homogeneous and heterogeneous bidders exhibit risk- and loss-aversion.[17] Moreover, the model with risk- and loss-averse bidders fits the data significantly better than the model with risk- and loss-neutral bidders or the model with risk-averse and loss-neutral bidders ($p < 0.01$, Likelihood Ratio tests).

6 Discussion

This experiment investigates individual bid timing decisions in all-pay auctions with complete information. Our results show that, compared to a LTB, an ETB rule induces more early bids for homogeneous bidders, especially in the early rounds. However, the use of different tie-breaking rules does not affect heterogeneous players' bid timing decisions. Particularly, we observe that high-cost bidders dominantly enter the late bidding stage.

Furthermore, when players have homogeneous abilities, the revenue in endogenous-entry treatments is not significantly different from that in exogenous-entry treatments.

[17] Both Klose and Sheremeta (2012) and Ernst and Thöni (2013) estimate $\alpha < 1$, while the size of λ varies with experiment conditions.

Table 5: Risk and Loss Aversion Estimation

	Homogeneous			High-Cost			Low-Cost		
		Model			Model			Model	
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
α	1	0.69	0.70	1	0.73	0.75	1	0.67	0.67
(σ^2)		(0.03)	(0.03)		(0.03)	(0.03)		(0.05)	(0.05)
λ	1	1	1.33	1	1	1.29	1	1	1.23
(σ^2)			(0.13)			(0.22)			(0.24)
-Log(L)	13304	13017	12982	5369	5280	5274	5126	4997	4989
LR Test (P-Values)									
1 v. 2			0.000			0.000			0.000
1 v. 3			0.000			0.000			0.000
2 v. 3			0.000			0.001			0.000
Obs.	2889	2889	2889	1306	1306	1306	1113	1113	1113

Conversely, for heterogeneous bidders, because of the low bids for high-cost bidders under ETB, their revenue is lower than in the exogenous-entry treatment.

Finally, we examine individual strategies in simultaneous all-pay auctions, and find that a significant proportion of bidders do not randomize as predicted by the risk-neutral model. Instead, bidders are more likely to have either extremely high or low bids. This finding is consistent with the predictions of an extended model with risk and loss aversion.

Our results have practical implications for contest designers, e.g., web designers for crowdsourcing contests. First, if the contest designer aims to attract earlier entrants, a simple ETB rule can be effective, especially for inexperienced entrants, who are the majority of users on platforms (Yang, Adamic and Ackerman 2008). Second, if they want to elicit more effort from participants, simultaneous contests would be a better choice than sequential ones.

However, many important and interesting features in real-life contest platforms are not considered in our study. For example, as participants dynamically enter contests, the number of contestants is unknown. Second, as contestants can be inspired by prior solutions and produce better solutions by free riding on prior solutions, it may be more effective to solicit high-quality solutions from sequential all-pay auctions than from simultaneous all-pay auctions. These constitute scope for future research.

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A Proofs

Equilibrium Characterization for All-Pay Auctions with n Players:

The analysis for heterogeneous bidders follows Konrad and Leininger (2007), and we focus on equilibrium characterization for homogeneous players. Assuming a favor-late tie-breaking rule, we first characterize the SPNE in sequential all-pay auctions.

Following Konrad and Leininger (2007), we first consider the subgame at the late stage (L). The maximum bids from the early stage (E) are characterized by $\bar{x}_E \equiv \max_{i \in E}\{x_i\}$. x_L^i represents the best response in stage L for bidder i. Given $\bar{x}_E$, we have:

1. If $c\bar{x}_E > v$, $x_L^i = 0$.
2. If $c\bar{x}_E = v$, if there is only one player in the late stage, she is indifferent between bidding v and 0. If there are more than one bidder, either all of them bid 0 or one of them bids v.
3. If $c\bar{x}_E < v$, if there is only one player in the late stage, she would bid $\bar{x}_E$. If there are more than one bidder, it becomes a simultaneous all-pay auction with lower bound $\bar{x}_E$, and the expected payoff for all late player is always 0.

Now we consider stage E. As $\bar{x}_L \equiv \max_{i \in L}\{\bar{x}_i\}$ and $\bar{x}_i = \bar{x} = \frac{v}{c}$ for homogeneous bidders, the equilibrium bids in stage E for bidder i: x_E^i, is characterized below.

1. If there is more than one bidder in stage E, there are two types of equilibrium strategies.
   (a) One player bids $\bar{x}$ and all others bid 0.
   (b) All players bid 0.
2. If there is one and only one bidder i in stage E, similarly, $x_E^i = \{\bar{x}, 0\}$.

When all bidders enter either stage E or stage L together, the game becomes a simultaneous all-pay auction and the unique symmetric Nash equilibrium is that bidders randomize continuously on $[0, \bar{x}]$. The expected payoff for everyone is always 0.

Furthermore, when the bid timing decision is endogenous, conditional on the subgame where all early players bid 0 in the equilibrium, $q_i^* = 0$ is a best reply if $\prod q_{j \neq i} > 0$. Otherwise, $q_i^* \in [0, 1]$.

∎

Proof of Proposition 1:

The proof for the unique symmetric NE follows the line of the argument in Baye et al. (1996), and we focus on the analysis of $G_1(x)$ and $G_2(x)$. The proof for homogeneous bidders is similar to $G_1(x)$, so we omit it.

As the expected payoff for bidder 2 is always 0 in the equilibrium (Siegel 2009), we obtain:

$$U_2(x) = G_1(x)u(v-x) + (1 - G_1(x))u(-x) = 0.$$

Consequently,

$$G_1(x) = \frac{-u(-x)}{u(v-x) - u(-x)}.$$

The utility function is defined below:

$$u(x) = \begin{cases} x^\alpha & \text{if } x \geq 0 \\ -\lambda(-x)^\alpha & \text{otherwise.} \end{cases}$$

As $G_1(\frac{v}{2})u(\frac{v}{2}) + (1 - G_1(\frac{v}{2}))u(-\frac{v}{2}) = 0$, $G_1(\frac{v}{2}) = \frac{\lambda}{1+\lambda} \geq \frac{1}{2}$.

As $\forall x \in (0, v)$, $U_2(x) = 0$ in the equilibrium, $U_2'(x) = 0$ and $U_2''(x) = 0$. Defining $g_1(x) = G_1'(x)$, we obtain:

$$g_1(x) = \frac{(1 - G_1(x))u'(-x) + G_1(x)u'(v-x)}{u(v-x) - u(-x)} > 0.$$

Furthermore, $U_2''(x) = 0$ yields the following equation:

$$g_1'(x)(u(v-x) - u(-x)) = 2g_1(x)(u'(v-x) - u'(-x)) - G_1(x)u''(v-x) - (1 - G_1(x))u''(-x).$$

Defining $Z(x) = 2g_1(x)(u'(v-x) - u'(-x)) - G_1(x)u''(v-x) - (1 - G_1(x))u''(-x)$, we show that $Z(x) \leq 0$ when $0 \leq x \ll v$, and that $Z(x) \geq 0$ when $0 \leq v - x \ll v$.

First, $\exists\ \frac{v}{2} \leq x_1 = \frac{v}{1+\lambda^{\frac{1}{\alpha-1}}} < v$, $u'(v - x_1) = u'(-x_1)$. As $u(x)$ is concave with $x \geq 0$, $\forall$ $x \geq x_1$, $u'(v-x) \geq u'(-x) > 0$, and by explicit calculation, $u''(v-x) \leq -u''(-x) < 0$. Together with $G_1(x) \geq \frac{1}{2}$, $Z(x) \geq 0$.

Second, as $G_1(\frac{v}{2}) \geq \frac{1}{2}$, then $\exists\ x_0 \leq \frac{v}{2}$, $G(x_0) = \frac{1}{2}$. Furthermore, $\forall\ x \leq x_0 < x_1$, $0 < u'(v - x) \leq u'(-x)$, and by explicit calculation, $-u''(-x) \leq u''(v-x) < 0$. Together with $G_1(x) \leq \frac{1}{2}$, $Z(x) \leq 0$.

Because of the continuity of $Z(x)$, $\exists\ y$ where $x_0 \leq y \leq x_1$, $Z(y) = 0$ and $g_1'(y) = 0$. Now we show that y is unique.

If $\exists\ Z(y_1) = Z(y_2) = 0$, by the continuity of $Z(x)$ and the intermediate value theorem, one of them, e.g., y_1, must have $Z'(y_1) \leq 0$. However, we know that

$$\begin{aligned} Z'(x) &= 2g_1'(x)(u'(v-x) - u'(-x)) + 2g_1(x)(-u''(v-x) + u''(-x)) \\ &-g_1(x)u''(v-x) + G_1(x)u'''(v-x) + g_1(x)u''(-x) + (1 - G_1(x))u'''(-x) \end{aligned}$$

When $x = y_1$, $Z'(y_1) = 3g_1(y_1)(u''(-y_1) - u''(v - y_1)) + G_1(y_1)u'''(v - y_1) + (1 - G_1(y_1))u'''(-y_1) > 0$, which is a contradiction.

Along the same line of proof for $G_1(x)$, $G_2''(x) < 0$ with $0 \leq x \ll v$, and $G_2''(x) > 0$ with $0 \leq v - x \ll v$. Additionally, as $G_2(0) = \frac{u((1-c_1)v)}{u(v)}$ and $G_2^{rn}(0) = \frac{(1-c_1)v}{v}$, by the concavity of $u(x)$ with $x \geq 0$, $G_2(0) \geq G_2^{rn}(0)$.

∎

B Experimental Instruction

This is an experiment in decision-making. The experiment will proceed in two parts and you will make a series of decisions in each part. At the end, you will fill out a post-experiment questionnaire.

This experiment has 12 participants. Each of you has been randomly assigned an experiment ID at the beginning of the experiment. The experimenter will use this ID to pay you at the end of the experiment.

Rounds: The experiment consists of 30 rounds of two-person auctions.

Endowment: Each of you has 125 tokens as an endowment at the beginning of each round.

Prize Values: At the beginning of each round, an object with a value of 100 tokens will be auctioned within each two-person group.

Matching: At the beginning of each round, you will be randomly matched with another person. You are equally likely to be matched with any other person in the room.

Decisions: In each round, you must make two decisions. First, both you and your match choose independently and simultaneously which bidding stage you want to enter. The entry decisions are then announced to both of you. Second, you and your match each choose a bid in your respective bidding stage.

Bids: There are two bidding stages: the early stage and the late stage.

1. If you enter early and your match enters late, you will choose a bid first. After observing your bid, your match will choose his or her bid.
2. If you enter late and your match enters early, your match will choose a bid first. After observing his or her bid, you will choose your own bid.
3. If both of you choose the same stage, you will bid simultaneously.

Cost of the Bid: The cost of the bid captures the idea that it is sometimes more or less costly to submit a bid. In the experiment, it is determined by a random number generator at the beginning of each round. For each round, with 50% chance, the cost of the bid is 1 token for you and 0.8 tokens for your match. With 50% chance, the cost of the bid is 0.8 tokens for you and 1 token for your match. Here is a numerical example:

1. If the cost of your bid is 1 token and you bid 50, then you will pay 50 tokens.
2. If the cost of your bid is 0.8 tokens and you bid 50, then you will pay 40 tokens.

Bid Range: Your bid can be any integer between 0 and 125, inclusive.

Profits: In each round, your profits will be determined by (1) your bid; (2) your match's bid; (3) the cost of your bid; and (4) the entry decisions in the event of a tie.

Profits= Your Endowment- the cost of your bid*your bid + the value of the object if you win = 125 - the cost of your bid*your bid+ 100 if you win

For example, in a given round, if you bid 40 and the cost of your bid is 0.8 in this round, then

1. If you win the auction, then your profit is $125 - 0.8 * 40 + 100 = 193$ tokens
2. If you lose the auction, then your profit is $125 - 0.8 * 40 = 93$ tokens

Note: You will always pay for your bid, which is equal to the cost of your bid* your bid, no matter whether you win or lose.

The tie-breaking rule: If you and your match bid exactly the same amount, and

1. Both of you enter in the same stage, we will randomly choose one as the winner.
2. If one and only one of you enter in the early stage, then the early bidder will be the winner.

Cumulative Profits: Your cumulative profits will be the sum of your profits in all rounds.

Feedback: At the end of each round, you will get the following feedback on your screen:

1. Your entry decision
2. Your match's entry decision
3. Your bid
4. Your match's bid
5. Your profits
6. Your match's profits
7. Your cumulative profits

History: In each round, your and your matches' bids and entry decisions in each previous round, your and your matches' profits in each previous round, as well as your cumulative profits up until the last round will be displayed in a history box.

Review Questions: To help you understand the experiment, we will go over nine review questions before we start the auction. You can also find these review questions in the appendix for your reference. You will get 25 tokens for answering each of the review questions correctly.

Exchange Rate: $1 = 250 tokens.

Please do not communicate with each other during the experiment. If you have a question, feel free to raise your hand, and an experimenter will come to help you.

Online Appendices

Lottery Choice

After 30 rounds of auctions, you will make choices for three series of paired lotteries, such as those represented as “Option A” and “Option B” below. In both series 1 and 2, there are 14 lottery questions. In series 3, there are 7 lottery questions.

For each series, you are asked to choose a “switch” question from Option A to Option B. For example, you can choose to switch from Option A to Option B in Question 6, which means you will choose Option A from Question 1 to 5 while you will choose Option B from Question 6 to the end of the series. You can also choose to never switch to Option B, which means you always choose Option A for all questions in the series. You can also choose to switch to Option B from Question 1, which means you always choose Option B for all questions in this series.

Even though there are 35 lottery questions, only one of them will end up being used. The selection of the one to be used depends on a random number generator, which is the equivalent of throwing a 35-sided die. Each lottery (Series 1: 1-14; Series 2: 1-14; Series 3: 1-7) is equally likely to be chosen.

After the lottery question is chosen, the money prize that you receive is determined by another random number generator, which is equivalent of throwing a ten-sided die. Each outcome, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10, is equally likely to be chosen. For example, if the number drawn from the first random number generator is 29, then the 29th lottery, which is the first lottery in series 3, is chosen. Furthermore, consider if the number drawn from the second random number generator is 4. If you chose Question 2 as the switch question for series 3, which means you choose Option A for question 1, then you will get 25 tokens. If you chose Question 1 as the switch question for series 3, which means you chose Option B for Question 1, you will get 30 tokens.

Review Questions: To help you understand the lottery, we will go over one review question before we start. You will also get 25 tokens for answering the review question correctly.

Recall the exchange rate is still $1 = 250 tokens in the lottery.

Final Payment: Your final payment in this experiment will be:

Your Earnings in the Review Question Part (for both auction and lottery) + Your Cumulative Profits in the Auction (Part1) + Your Profits in the lottery (Part2) + Participation Fee ($ 5)

Series1-Lottery Number	Option A	Option B
1	40 if the die is 1-3 10 if the die is 4-10	68 if the die is 1 5 if the die is 2-10
2	40 if the die is 1-3 10 if the die is 4-10	75 if the die is 1 5 if the die is 2-10
3	40 if the die is 1-3 10 if the die is 4-10	83 if the die is 1 5 if the die is 2-10
4	40 if the die is 1-3 10 if the die is 4-10	93 if the die is 1 5 if the die is 2-10
5	40 if the die is 1-3 10 if the die is 4-10	106.5 if the die is 1 5 if the die is 2-10
6	40 if the die is 1-3 10 if the die is 4-10	125 if the die is 1 5 if the die is 2-10
7	40 if the die is 1-3 10 if the die is 4-10	150 if the die is 1 5 if the die is 2-10
8	40 if the die is 1-3 10 if the die is 4-10	185 if the die is 1 5 if the die is 2-10
9	40 if the die is 1-3 10 if the die is 4-10	220 if the die is 1 5 if the die is 2-10
10	40 if the die is 1-3 10 if the die is 4-10	300 if the die is 1 5 if the die is 2-10
11	40 if the die is 1-3 10 if the die is 4-10	400 if the die is 1 5 if the die is 2-10
12	40 if the die is 1-3 10 if the die is 4-10	600 if the die is 1 5 if the die is 2-10
13	40 if the die is 1-3 10 if the die is 4-10	1000 if the die is 1 5 if the die is 2-10
14	40 if the die is 1-3 10 if the die is 4-10	1700 if the die is 1 5 if the die is 2-10

Series2-Lottery Number	Option A	Option B
1 (15)	40 if the die is 1-9 30 if the die is 10	54 if the die is 1-7 5 if the die is 8-10
2 (16)	40 if the die is 1-9 30 if the die is 10	56 if the die is 1-7 5 if the die is 8-10
3 (17)	40 if the die is 1-9 30 if the die is 10	58 if the die is 1-7 5 if the die is 8-10
4 (18)	40 if the die is 1-9 30 if the die is 10	60 if the die is 1-7 5 if the die is 8-10
5 (19)	40 if the die is 1-9 30 if the die is 10	62 if the die is 1-7 5 if the die is 8-10
6 (20)	40 if the die is 1-9 30 if the die is 10	65 if the die is 1-7 5 if the die is 8-10
7 (21)	40 if the die is 1-9 30 if the die is 10	68 if the die is 1-7 5 if the die is 8-10
8 (22)	40 if the die is 1-9 30 if the die is 10	72 if the die is 1-7 5 if the die is 8-10
9 (23)	40 if the die is 1-9 30 if the die is 10	77 if the die is 1-7 5 if the die is 8-10
10 (24)	40 if the die is 1-9 30 if the die is 10	83 if the die is 1-7 5 if the die is 8-10
11 (25)	40 if the die is 1-9 30 if the die is 10	90 if the die is 1-7 5 if the die is 8-10
12 (26)	40 if the die is 1-9 30 if the die is 10	100 if the die is 1-7 5 if the die is 8-10
13 (27)	40 if the die is 1-9 30 if the die is 10	110 if the die is 1-7 5 if the die is 8-10
14 (28)	40 if the die is 1-9 30 if the die is 10	130 if the die is 1-7 5 if the die is 8-10

Series3-Lottery Number	Option A	Option B
1 (29)	25 if the die is 1-5 -4 if the die is 6-10	30 if the die is 1-5 -21 if the die is 6-10
2 (30)	4 if the die is 1-5 -4 if the die is 6-10	30 if the die is 1-5 -21 if the die is 6-10
3 (31)	1 if the die is 1-5 -4 if the die is 6-10	30 if the die is 1-5 -21 if the die is 6-10
4 (32)	1 if the die is 1-5 -4 if the die is 6-10	30 if the die is 1-5 -16 if the die is 6-10
5 (33)	1 if the die is 1-5 -8 if the die is 6-10	30 if the die is 1-5 -16 if the die is 6-10
6 (34)	1 if the die is 1-5 -8 if the die is 6-10	30 if the die is 1-5 -14 if the die is 6-10
7 (35)	1 if the die is 1-5 -8 if the die is 6-10	30 if the die is 1-5 -11 if the die is 6-10

Post-questionnaire

We are interested in whether there is a correlation between participants' decision-making behavior and certain socio-psychological factors. The following information will be very helpful for our research. This information will be strictly confidential.

1. Gender
   (a) Male
   (b) Female
2. Ethnic Background
   (a) White
   (b) Asian / Asian American
   (c) African American
   (d) Hispanic
   (e) Native American
   if it is other, please specify:______________________
3. Age
4. How many siblings do you have?
5. Grad/Year
   (a) Freshman
   (b) Sophomore
   (c) Junior
   (d) Senior
   (e) $>$ 4 years
   (f) Graduate student
6. Major
7. From which countries did your family originate?
8. Would you describe yourself as (Please choose one)
   (a) Optimistic
   (b) Pessimistic
   (c) Neither
9. Which of the following emotions did you experience during the experiment? (You may choose any number of them.)
   (a) Anger
   (b) Anxiety
   (c) Confusion
   (d) Contentment
   (e) Fatigue
   (f) Happiness
   (g) Irritation
   (h) Mood swings
   (i) Withdrawal
10. In general, do you see yourself as someone who is willing, even eager, to take risks? (1-7 likert scale)
11. Concerning just personal finance decisions, do you see yourself as someone who is willing, even eager, to take risks?
12. In general, do you see yourself as someone who, when faced with an uncertain situation, worries a lot about possible losses?
13. Concerning just personal finance decisions, are you someone who, when faced with an uncertain situation, worries a lot about possible losses?
14. In general, how competitive do you think you are?
15. Concerning just sports and leisure activities, how competitive do you think you are?
    How much do you agree with the following statements? (1-7 likert scale)
    I see myself as someone who
16. is helpful and unselfish with others
17. can be cold and aloof
18. is considerate and kind to almost everyone
19. likes to cooperate with others

20. is often on bad terms with others
21. feels little concern for others
22. is on good terms with nearly everyone
23. can make my own decisions, uninfluenced by public opinion.
24. It is achievement, rather than popularity with others, that gets you ahead nowadays.
25. I will stick to my opinion if I think I am right, even if others disagree.
26. I will change the opinion I express as a result of an onslaught of criticism, even though I really do not change the way I feel.
27. The important thing in being successful nowadays is not how hard you work, but how well you fit in with the crowd.
28. I am more likely to express my opinion in a group when I see others agree with me.
29. In a given round, you have 125 tokens in your deposit account and you lose 25 of them after participating in the auction. How much money do you think you win or lose? I.e. given that you have 125 tokens in your deposit account already, do you consider it a loss if you end the auction with fewer than your original 125 tokens, or only if you lose the entirety of your endowment?
    (a) I lost 25; I consider ending the auction with any amount less than the original 125 tokens in my personal account to be a loss.
    (b) I won 100 tokens; I only consider the auction's outcome a loss if I lose the entirety of my 125-token endowment.
30. If you chose to enter the early bidding stage in any round, please write down the reason (sequential all-pay treatments only)
31. If you chose to enter the late bidding stage in any round, please write down the reason (sequential all-pay treatments only)

Testing Mixed Strategies in Simultaneous All-Pay Auctions

Following Wooders (2010), we categorize homogeneous bidders' bids in simultaneous all-pay auctions on [0, 100] into two categories: (1) [0,50]: representing low bids and (2) (50,100]: representing high bids. 50 is the expected average bid under the Nash equilibrium predictions. Table 6 shows the bidding counts for each category and the corresponding randomized binomial test results for homogeneous bidders.[18] With a null hypothesis that the probability for a bidder to choose a bid of less than 50 is 0.5, we find more rejection numbers than expected for each treatment.[19] Specifically, the null hypothesis that bidders choose low bids with probability 0.5 is rejected at the 5% level for 21 out of 36 bidders in exogenous-entry treatments, 22 rejections in ETB treatments, and 24 in LTB treatments.[20]

To test the joint null hypothesis that all bidders in each treatment are equally likely to bid between these two different bidding categories, we examine the empirical distribution of the 36 p-values for the low bids from random binomial tests in each treatment. Under the null hypothesis that bidders choose low bids with probability 0.5, the p-value should be uniformly distributed in [0,1] for each treatment. The left column of Figure 11 presents the empirical CDF of p-values in each treatment. The Kolmogorov-Smirnov (KS) test shows that none of these distributions is uniform ($p < 0.01$).

Furthermore, we check the serial independence of bids by applying the method outlined in Walker and Wooders (2001). The null hypothesis is that each bid between low and high is serially independent. We reject the null hypothesis if there are too many or too few runs.[21] Tables 7, 8, and 9 report the data and results for the serial independence test. $F(r)$ denotes the probability of obtaining r, or few, runs. The null hypothesis is rejected at the 5% level if $F(r) < 0.025$ or $1 - F(r-1) < 0.025$. In summary, there are 17 out of 36 rejections in exogenous-entry treatments, 14 rejections in ETB treatments, and 17 rejections in LTB treatments. In particular, these rejections occur because there are too few runs, $F(r) < 0.025$, indicating that bidders keep over or underbidding. In addition, to test the joint null hypothesis that bidders are serially independent in each treatment, we construct a statistic t^i by randomly drawing a number from the uniform distribution U[F(r-1),F(r)]. A particular realization of this statistic is given in the right column of Tables 7, 8, and 9. Under the null hypothesis of serial independence, t^i is uniformly distributed in [0,1]. The right column of Figure 11 presents the empirical CDF of the realized values in each treatment. Kolmogorov-Smirnov (KS) tests show that these distributions are not uniform ($p < 0.01$).

For high-cost and low-cost bidders, we apply the same technique to examine whether individuals play a mixed strategy as predicted by a risk-neutral model. We categorize individual bids on [0, 100] into two categories. As the average bid predicted by the Nash equilibrium for high (low)-cost bidders is 40 (50), we use 40 as the cutoff for high-cost bidders and 50 for low-cost bidders. Tables 10 and 14 show bidding counts for each category and the corresponding randomized binomial test result for each bidder. For high-cost bidders, the null hypothesis that bidders choose low bids with probability 0.52 is rejected at the 5% level for 16 out of 36 bidders in exogenous-entry treatments,[22] 12 bidders in the ETB treatments, and 12 bidders in LTB treatments. For low-cost bidders, the null hypothesis that bidders choose low bids with probability 0.5 is rejected at the 5% level for 18 out of 36 bidders in exogenous-entry treatments, 9 bidders in the ETB treatments, and 10 bidders in the LTB treatments. The Kolmogorov-Smirnov (KS) test also rejects the joint null hypothesis that high(low)-cost bidders in each treatment bid low bids with probability 0.52 (0.5). Figures 12 and 13 (left column) present the empirical CDF of p-values for high- and low-cost bidders in each treatment. In addition, regarding serial independence, for high-cost bidders, there are 8 rejections in exogenous-entry treatments, 12 rejections in ETB treatments, and 12 rejections in LTB treatments. For low-cost bidders, there are 15 rejections in exogenous-entry treatments, 6 rejections in ETB treatments, and 6 rejections in LTB treatments. Tables 11, 12, 13, 15, 16, and 17 report the data and results for the test of serial independence. Kolmogorov-Smirnov (KS) tests also reject the joint null hypothesis that bidders are serially independent in each treatment. Figures 12 and 13 (right column) present

18 As bids above 100 are not predicted by the NE and an expected probability of each type of bid is required in a binomial test, we exclude this category in our binomial tests. The proportion of bids above 100 is 4% for homogeneous bidders, 12% for high-cost bidders and 25% for low-cost bidders.

19 The expected rejection number is 1.8 bidders ($36 \times 5\% = 1.8$).

20 As the probability that bidders choose high bids is also 0.5, we obtain identical numbers of rejections for high bids.

21 A run is a maximal string of consecutive identical symbols, either all low bids or high bids. For example, the bidding sequence s={L,L,H,L} has three runs.

22 There is a mass point at bid 0 with probability 0.2 for high-cost bidders.

the empirical CDF of the realized values of t^i in each treatment for both high- and low-cost bidders and they are significantly different from uniform distributions ($p < 0.01$).

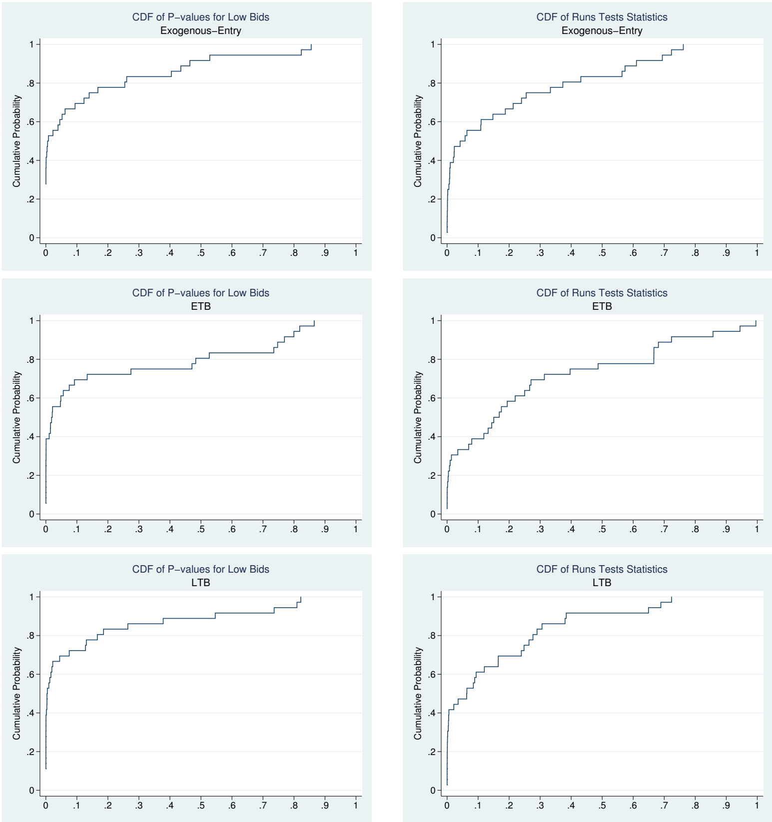

Figure 11: Randomized Binomial Tests and Runs Tests in Simultaneous All-Pay Auctions for Homogeneous Bidders

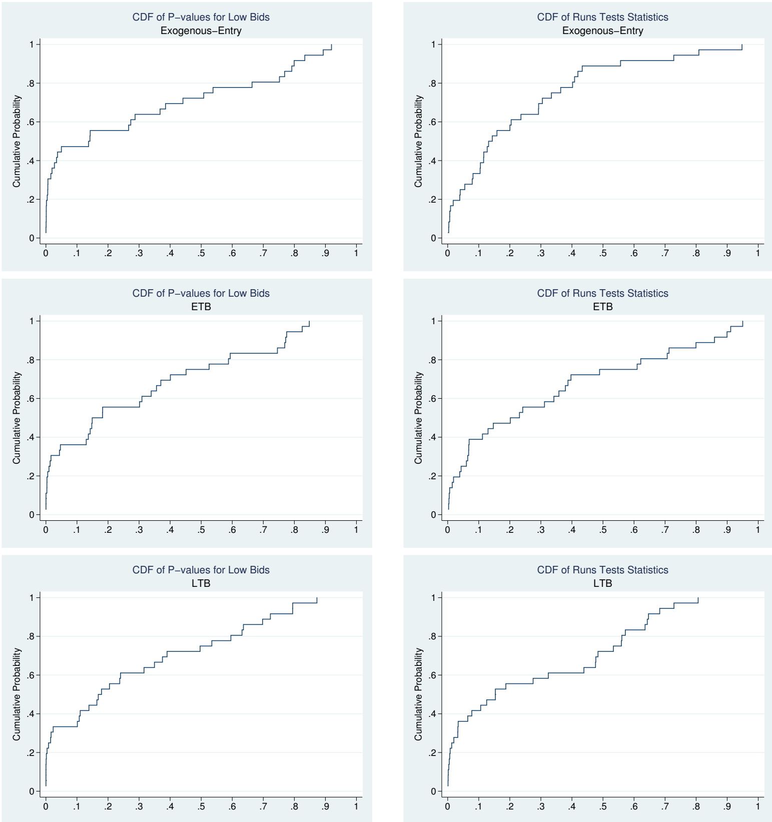

Figure 12: Randomized Binomial Tests and Runs Tests in Simultaneous All-Pay Auctions for High-Cost Bidders

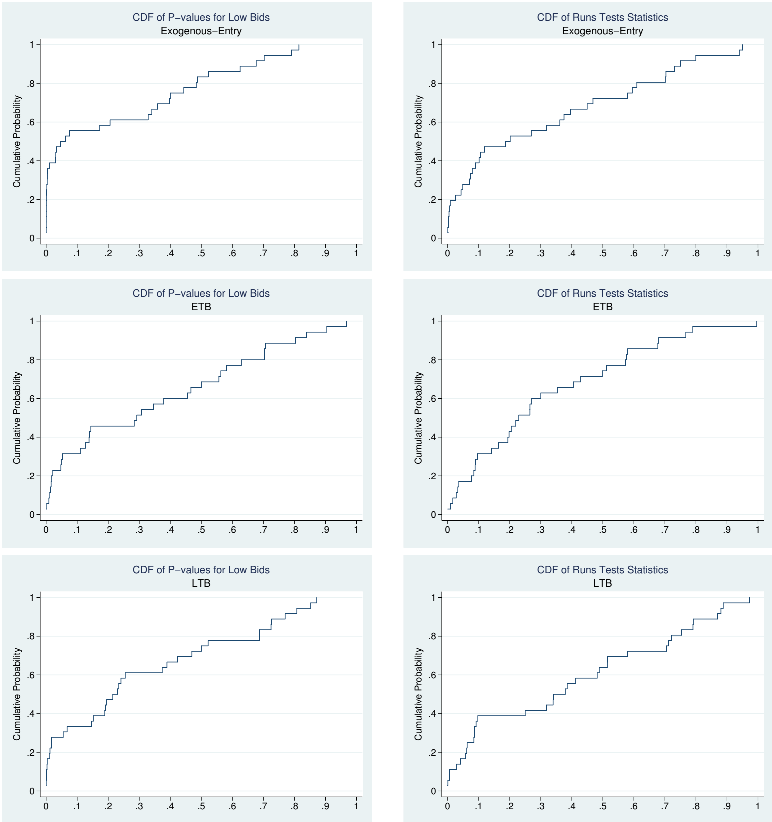

Figure 13: Randomized Binomial Tests and Runs Tests in Simultaneous All-Pay Auctions for Low-Cost Bidders

Table 6: Bids for Homogeneous Bidders in Simultaneous All-Pay Auctions

Treatment		Exogenous-Entry		Endogenous-Entry-ETB		Endogenous-Entry-LTB	
Session	Subject	[0,50]	(50,100]	[0,50]	(50,100]	[0,50]	(50,100]
1	1	7**	23**	13	15	24**	4**
	2	20	9**	13	14	18	12
	3	14	7	23 **	6 **	30 **	0 **
	4	0**	30**	10	19	29 **	1 **
	5	20	10	26 **	1 **	24 **	6 **
	6	23**	7**	23 **	5 **	16	12
	7	21**	7**	28 **	1 **	7 **	23 **
	8	2**	28**	12	15	23 **	6 **
	9	18	10	13	14	9	17
	10	26**	4**	24 **	4 **	20 **	5 **
	11	24**	0**	16	9	30 **	0 **
	12	26**	4**	3 **	25 **	29 **	1 **
2	1	7**	18**	8 **	19 **	14	12
	2	13	17	21 **	8 **	21 **	8 **
	3	18	10	8	12	12	15
	4	3**	16**	21 **	4 **	19	10
	5	29**	1**	23 **	5 **	22 **	7 **
	6	16	11	12	16	3 **	24 **
	7	22**	7**	19 **	8	6 **	21 **
	8	3**	27**	5 **	17 **	21 **	9 **
	9	10	10	12	15	1 **	15 **
	10	13	17	8 **	18 **	4 **	25 **
	11	16	12	1 **	20 **	13	15
	12	5**	25**	12	10	19	11
3	1	29**	1**	4 **	23 **	24 **	5**
	2	9	15	17	7	3**	26**
	3	15	15	11	10	4 **	25**
	4	8	18**	3 **	24 **	21 **	8 **
	5	21**	9**	0 **	24 **	19	9
	6	29**	1**	3 **	0	19	10
	7	3**	17**	5**	16 **	24**	1**
	8	18	12	24**	0**	7**	20**
	9	11	19	9**	1**	7**	21**
	10	0**	21**	23**	0**	5**	23**
	11	23**	5**	10	12	15	14
	12	25**	5**	7	15	11	18

Table 7: Runs Tests for Homogeneous Bidders in Exogenous-Entry Treatments

Session	Subject	[0,50]	(50,100]	Runs	F(r-1)	F(r)	U[F(r-1),F(r)]
1	1	7	23	6	0.001	0.004**	0.002
	2	20	9	8	0.005	0.016**	0.010
	3	14	7	4	0.000	0.002**	0.001
	4	0	30	1	0.000	1**	0.993
	5	20	10	10	0.022	0.055	0.038
	6	23	7	8	0.018	0.048	0.034
	7	21	7	12	0.482	0.639	0.591
	8	2	28	2	0.000	0.005**	0.004
	9	18	10	15	0.536	0.661	0.569
	10	26	4	7	0.108	0.371	0.310
	11	24	0	1	0.000	1**	0.421
	12	26	4	2	0.000	0.000**	0.000
2	1	7	18	1	0.363	0.393	0.375
	2	13	17	19	0.632	0.608	0.621
	3	18	10	12	0.139	0.258	0.156
	4	3	16	3	0.002	0.020**	0.007
	5	29	1	2	0.000	0.067	0.035
	6	16	11	11	0.076	0.124	0.114
	7	22	7	11	0.259	0.459	0.456
	8	3	27	5	0.033	0.2	0.196
	9	10	10	13	0.673	0.731	0.676
	10	13	17	7	0.000	0.001**	0.001
	11	16	12	6	0.000	0.000**	0.000
	12	5	25	5	0.002	0.010**	0.002
3	1	29	1	3	0.067	1	0.536
	2	9	15	6	0.001	0.005**	0.004
	3	15	15	14	0.123	0.24	0.172
	4	8	18	8	0.017	0.047	0.037
	5	21	9	4	0.000	0.000**	0.000
	6	29	1	3	0.067	1	0.109
	7	3	17	6	0.298	0.509	0.427
	8	18	12	10	0.011	0.029	0.019
	9	11	19	3	0.000	0.000**	0.000
	10	0	21	1	0.000	1**	0.763
	11	23	5	5	0.002	0.013**	0.002
	12	25	5	5	0.002	0.010**	0.002

Table 8: Runs Tests for Homogeneous Bidders in ETB Treatments

Session	Subject	[0,50]	(50,100]	Runs	F(r-1)	F(r)	U[F(r-1),F(r)]
1	1	13	15	12	0.064	0.149	0.065
	2	13	14	8	0.002	0.009**	0.006
	3	23	6	5	0.001	0.003**	0.003
	4	10	19	9	0.010	0.028	0.015
	5	26	1	3	0.000	0.926	0.588
	6	23	5	3	0.000	0.000**	0.000
	7	28	1	3	0.069	1	0.548
	8	12	15	5	0.000	0.000**	0.000
	9	13	14	13	0.189	0.248	0.238
	10	24	4	6	0.049	0.123	0.090
	11	16	9	5	0.000	0.214	0.024
	12	3	25	5	0.038	0.214	0.135
2	1	8	19	13	0.388	0.415	0.411
	2	21	8	9	0.028	0.077	0.075
	3	8	12	8	0.067	0.159	0.082
	4	21	4	9	0.617	1	1.000
	5	23	5	2	0.000	0.000**	0.000
	6	12	16	14	0.206	0.358	0.349
	7	19	8	11	0.197	0.334	0.300
	8	5	17	9	0.398	0.696	0.410
	9	12	15	6	0.000	0.001**	0.000
	10	8	18	7	0.005	0.017**	0.008
	11	1	20	3	0.095	1	0.887
	12	12	10	10	0.142	0.271	0.168
3	1	4	23	2	0.000	0.000**	0.000
	2	17	7	11	0.397	0.591	0.531
	3	11	10	13	0.606	0.681	0.660
	4	3	24	7	0.395	1	0.828
	5	0	24	1	0.000	1**	0.607
	6	3	0	1	0.000	1**	0.000
	7	5	16	4	0.001	0.007**	0.003
	8	24	0	1	0.000	1**	0.521
	9	9	1	2	0.000	0.2	0.008
	10	23	0	1	0.000	1**	0.546
	11	10	12	10	0.142	0.271	0.231
	12	7	15	6	0.006	0.022**	0.015

Table 9: Runs Tests for Homogeneous Bidders in LTB Treatments

Session	Subject	[0,50]	(50,100]	Runs	F(r-1)	F(r)	U[F(r-1),F(r)]
1	1	24	4	9	0.568	1	0.871
	2	18	12	4	0.000	0.000**	0.000
	3	30	0	1	0.000	1**	0.914
	4	29	1	2	0.000	0.067	0.041
	5	24	6	2	0.000	0.000**	0.000
	6	16	12	8	0.002	0.007**	0.005
	7	7	23	5	0.000	0.001**	0.001
	8	23	6	7	0.013	0.05	0.031
	9	9	17	5	0.000	0.001**	0.000
	10	20	5	8	0.179	0.325	0.257
	11	30	0	1	0.000	1**	0.921
	12	29	1	3	0.067	1	0.873
2	1	14	12	11	0.085	0.129	0.104
	2	21	8	7	0.002	0.009**	0.002
	3	12	15	9	0.010	0.026	0.024
	4	19	10	8	0.003	0.010**	0.003
	5	22	7	9	0.055	0.144	0.132
	6	3	24	6	0.222	0.395	0.343
	7	6	21	10	0.322	0.486	0.406
	8	21	9	11	0.085	0.161	0.092
	9	1	15	3	0.125	1	0.446
	10	4	25	4	0.001	0.007**	0.002
	11	13	15	6	0.000	0.000**	0.000
	12	19	11	3	0.000	0.000**	0.000
3	1	24	5	8	0.135	0.254	0.178
	2	3	26	7	0.371	1	0.503
	3	4	25	2	0.000	0.000**	0.000
	4	21	8	6	0.000	0.002**	0.001
	5	19	9	6	0.000	0.002**	0.001
	6	19	10	13	0.228	0.345	0.315
	7	24	1	3	0.080	1	0.728
	8	7	20	5	0.000	0.002**	0.000
	9	7	21	11	0.281	0.306	0.294
	10	5	23	10	0.583	0.732	0.590
	11	15	14	12	0.048	0.114	0.061
	12	11	18	6	0.000	0.000**	0.000

Table 10: Bids for High-Cost Bidders in Simultaneous All-Pay Auctions

Treatment		Exogenous-Entry		Endogenous-Entry-ETB		Endogenous-Entry-LTB	
Session	Subject	[0,40]	(40,100]	[0,40]	(40,100]	[0,40]	(40,100]
1	1	4	0**	4	4	3	8
	2	10**	2**	12**	0**	12	5
	3	6	6	7	6	3	5
	4	7	13	4	5	8	4
	5	8	7	9	3	4	3
	6	3	6	12**	2**	5	3
	7	11**	0**	14**	4**	11**	0**
	8	4	5	9**	0**	8	2
	9	14**	3**	12	4	7	3
	10	14**	1**	9	5	0**	13**
	11	8	5	10	7	6	5
	12	4	4	14**	5	10	4
2	1	2**	10**	12	5	12**	1**
	2	8	8	12**	0**	10	3
	3	11**	1**	6	5	9	4
	4	12**	3**	2	3	5	5
	5	11	5	2	3	3	5
	6	0**	11**	10**	1**	8	9
	7	5	9	1	0	0**	13**
	8	9	8	4	1	1**	12**
	9	10	6	6	3	14**	3**
	10	6	8	9**	0**	6	3
	11	13**	1**	9**	0**	5	9
	12	4	8	3**	12**	1**	8**
3	1	1**	9**	1	1	5	5
	2	4	11	9	3	7	3
	3	3**	12**	1	5	3	3
	4	14**	1**	13**	2**	10	4
	5	1**	14**	11	4	15**	2**
	6	1	3	7	3	11**	2**
	7	10	8	2	6	0**	14**
	8	6	6	10	4	19**	0**
	9	11**	2**	8	8	4	5
	10	7	5	6	6	1	5**
	11	12**	1**	8	4	6	3
	12	18**	0**	17**	0**	12**	3**

Table 11: Runs Tests for High-Cost Bidders in Exogenous-Entry Treatments

Session	Subject	[0,40]	(40,100]	Runs	F(r-1)	F(r)	U[F(r-1),F(r)]
1	1	4	0	1	0.000	1**	0.614
	2	10	2	4	0.182	0.455	0.401
	3	6	6	3	0.002	0.013**	0.006
	4	7	13	5	0.002	0.010**	0.008
	5	8	7	6	0.051	0.149	0.137
	6	3	6	5	0.345	0.643	0.469
	7	11	0	1	0.000	1**	0.290
	8	4	5	4	0.071	0.262	0.117
	9	14	3	2	0.000	0.003**	0.001
	10	14	1	2	0.000	0.133	0.130
	11	8	5	8	0.576	0.793	0.602
	12	4	4	2	0.000	0.029	0.005
2	1	2	10	4	0.182	0.455	0.185
	2	8	8	6	0.032	0.100	0.060
	3	11	1	2	0.000	0.167	0.044
	4	12	3	4	0.033	0.130	0.082
	5	11	5	5	0.022	0.077	0.068
	6	0	11	1	0.000	1**	0.421
	7	5	9	3	0.001	0.007**	0.005
	8	9	8	9	0.319	0.500	0.404
	9	10	6	5	0.013	0.047	0.026
	10	6	8	9	0.646	0.821	0.764
	11	13	1	2	0.000	0.143	0.001
	12	4	8	5	0.109	0.279	0.246
3	1	1	9	3	0.200	1.000	0.961
	2	4	11	6	0.176	0.374	0.319
	3	3	12	4	0.033	0.130	0.082
	4	14	1	3	0.133	1.000	0.214
	5	1	14	2	0.000	0.133	0.073
	6	1	3	2	0.000	0.500	0.461
	7	10	8	8	0.117	0.251	0.156
	8	6	6	6	0.175	0.392	0.348
	9	11	2	4	0.167	0.423	0.308
	10	7	5	6	0.197	0.424	0.421
	11	12	1	2	0.000	0.154	0.073
	12	18	0	1	0.000	1**	0.109

Table 12: Runs Tests for High-Cost Bidders in ETB Treatments

Session	Subject	[0,40]	(40,100]	Runs	F(r-1)	F(r)	U[F(r-1),F(r)]
1	1	4	4	4	0.114	0.371	0.192
	2	12	0	1	0.000	1**	0.142
	3	7	6	8	0.500	0.733	0.584
	4	4	5	3	0.016	0.071	0.037
	5	9	3	2	0.000	0.009**	0.002
	6	12	2	4	0.154	0.396	0.184
	7	14	4	5	0.031	0.121	0.058
	8	9	0	1	0.000	1**	0.193
	9	12	4	9	0.819	1	0.933
	10	9	5	10	0.902	0.972	0.967
	11	10	7	9	0.355	0.549	0.468
	12	14	5	8	0.299	0.496	0.488
2	1	12	5	3	0.000	0.003**	0.002
	2	12	0	1	0.000	1**	0.579
	3	6	5	6	0.262	0.522	0.304
	4	2	3	2	0.000	0.2	0.028
	5	2	3	4	0.500	0.9	0.520
	6	10	1	2	0.000	0.182	0.153
	7	1	0	1	0.000	1**	0.866
	8	4	1	2	0.000	0.4	0.193
	9	6	3	4	0.107	0.345	0.115
	10	9	0	1	0.000	1**	0.686
	11	9	0	1	0.000	1**	0.960
	12	3	12	3	0.004	0.033	0.017
3	1	1	1	2	0.000	1	0.381
	2	9	3	2	0.000	0.009**	0.009
	3	1	5	2	0.000	0.333	0.217
	4	13	2	3	0.019	0.143	0.042
	5	11	4	6	0.176	0.374	0.365
	6	7	3	3	0.017	0.083	0.056
	7	2	6	5	0.643	1	0.949
	8	10	4	4	0.014	0.068	0.066
	9	8	8	6	0.032	0.1	0.065
	10	6	6	3	0.002	0.013**	0.008
	11	8	4	3	0.004	0.024**	0.016
	12	17	0	1	0.000	1**	0.954

Table 13: Runs Tests for High-Cost Bidders in LTB Treatments

Session	Subject	[0,40]	(40,100]	Runs	F(r-1)	F(r)	U[F(r-1),F(r)]
1	1	3	8	6	0.533	0.788	0.637
	2	12	5	4	0.003	0.017**	0.015
	3	3	5	5	0.429	0.714	0.696
	4	8	4	6	0.279	0.533	0.519
	5	4	3	2	0.000	0.057	0.046
	6	5	3	4	0.143	0.429	0.195
	7	11	0	1	0.000	1**	0.901
	8	8	2	5	0.533	1	0.688
	9	7	3	4	0.083	0.283	0.190
	10	0	13	1	0.000	1**	0.127
	11	6	5	6	0.262	0.522	0.423
	12	10	4	2	0.000	0.002**	0.000
2	1	12	1	3	0.154	1	0.437
	2	10	3	2	0.000	0.007**	0.004
	3	9	4	2	0.000	0.003**	0.002
	4	5	5	4	0.040	0.167	0.054
	5	3	5	5	0.429	0.714	0.585
	6	8	9	4	0.001	0.005**	0.005
	7	0	13	1	0.000	1**	0.588
	8	1	12	2	0.000	0.154	0.001
	9	14	3	6	0.350	0.579	0.535
	10	6	3	3	0.024	0.107	0.071
	11	5	9	4	0.007	0.039	0.028
	12	1	8	2	0.000	0.222	0.191
3	1	5	5	3	0.008	0.04	0.039
	2	7	3	4	0.083	0.283	0.270
	3	3	3	4	0.300	0.7	0.554
	4	10	4	2	0.000	0.002**	0.002
	5	15	2	2	0.000	0.015**	0.000
	6	11	2	2	0.000	0.026	0.001
	7	0	14	1	0.000	1**	0.367
	8	19	0	1	0.000	1**	0.694
	9	4	5	6	0.500	0.786	0.706
	10	1	5	3	0.333	1	0.998
	11	6	3	5	0.345	0.643	0.375
	12	12	3	4	0.033	0.13	0.050

Table 14: Bids for Low-Cost Bidders in Simultaneous All-Pay Auctions

Treatment		Exogenous-Entry		Endogenous-Entry-ETB		Endogenous-Entry-LTB	
Session	Subject	[0,50]	(50,100]	[0,50]	(50,100]	[0,50]	(50,100]
1	1	7**	0**	$bids > 100$		2**	9**
	2	1	3	4	12	4	3
	3	7	9	8	7	1	7
	4	2	5	8	8	3	6
	5	4	10	7	9	3	3
	6	15**	1**	5	8	14**	3**
	7	14**	0**	3	7	5	10
	8	8	2**	2	6	3	4
	9	13**	0**	4	5	1	5
	10	12**	0**	5	7	0	3**
	11	10	6	8	5	3	4
	12	0	1	4	5	7	6
2	1	12**	4**	1	1	11**	1**
	2	0**	6**	1	0	4	4
	3	4	9	2**	7	3**	12**
	4	13**	2**	2	3	3	3
	5	7	7	1	2	3	4
	6	6	8	5	3	5	2
	7	6	9	3	1	2	4
	8	1**	12**	9	2	13**	0**
	9	12**	2**	11	3	7	2
	10	1**	12**	6	3	7	13
	11	3**	10	8**	1**	10	3**
	12	4	5	9**	0**	1	2
3	1	7	4	4	7	8	5
	2	0**	13**	11**	3**	1	2
	3	2	2	0**	7**	1**	14**
	4	2**	13**	11**	2**	14**	1**
	5	1**	13**	4	9	4	2
	6	0**	15**	7	3	0**	6**
	7	7	5	8	7	0	2**
	8	9	9	11**	2**	1	1
	9	6	3	5	7	5	1
	10	6	4	12**	3**	1	3
	11	13**	4**	3	8	19**	0**
	12	10**	0**	11**	0**	11**	2**

Table 15: Runs Tests for Low-Cost Bidders in Exogenous-Entry Treatments

Session	Subject	[0,50]	(50,100]	Runs	F(r-1)	F(r)	U[F(r-1),F(r)]
1	1	7	0	1	0.000	1**	0.431
	2	1	3	3	0.500	1.000	0.696
	3	7	9	3	0.000	0.001**	0.000
	4	2	5	3	0.095	0.333	0.102
	5	4	10	6	0.203	0.419	0.386
	6	15	1	3	0.125	1.000	0.527
	7	14	0	1	0.000	1**	0.203
	8	8	2	3	0.044	0.222	0.155
	9	13	0	1	0.000	1**	0.306
	10	12	0	1	0.000	1**	0.288
	11	10	6	4	0.002	0.013**	0.004
	12	0	1	1	0.000	1**	0.240
2	1	12	4	6	0.154	0.335	0.303
	2	0	6	1	0.000	1**	0.623
	3	4	9	4	0.018	0.085	0.076
	4	13	2	4	0.143	0.371	0.199
	5	7	7	5	0.025	0.078	0.035
	6	6	8	8	0.413	0.646	0.499
	7	6	9	6	0.063	0.175	0.095
	8	1	12	2	0.000	0.154	0.112
	9	12	2	5	0.396	1.000	0.708
	10	1	12	3	0.154	1.000	0.850
	11	3	10	2	0.000	0.007**	0.005
	12	4	5	5	0.262	0.500	0.319
3	1	7	4	7	0.606	0.833	0.711
	2	0	13	1	0.000	1**	0.038
	3	2	2	2	0.000	0.333	0.197
	4	2	13	2	0.000	0.019**	0.002
	5	1	13	2	0.000	0.143	0.074
	6	0	15	1	0.000	1**	0.230
	7	7	5	2	0.000	0.003**	0.000
	8	9	9	6	0.012	0.044	0.018
	9	6	3	3	0.024	0.107	0.060
	10	6	4	2	0.000	0.010**	0.001
	11	13	4	4	0.007	0.037	0.008
	12	10	0	1	0.000	1**	0.208

Table 16: Runs Tests for Low-Cost Bidders in ETB Treatments

Session	Subject	[0,50]	(50,100]	Runs	F(r-1)	F(r)	U[F(r-1),F(r)]
1	1						
	2	4	12	4	0.009	0.045	0.025
	3	8	7	7	0.149	0.296	0.295
	4	8	8	6	0.032	0.1	0.096
	5	7	9	2	0.000	0.000**	0.000
	6	5	8	3	0.002	0.010**	0.003
	7	3	7	4	0.083	0.283	0.212
	8	2	6	3	0.071	0.286	0.204
	9	4	5	5	0.262	0.5	0.496
	10	5	7	7	0.424	0.652	0.576
	11	8	5	6	0.152	0.347	0.156
	12	4	5	4	0.071	0.262	0.204
2	1	1	1	2	0.000	1	0.604
	2	1	0	1	0.000	1**	0.670
	3	2	7	3	0.056	0.25	0.113
	4	2	3	2	0.000	0.2	0.051
	5	1	2	3	0.667	1	0.758
	6	5	3	5	0.429	0.714	0.711
	7	3	1	2	0.000	0.5	0.325
	8	9	2	2	0.000	0.036	0.023
	9	11	3	5	0.148	0.423	0.345
	10	6	3	4	0.107	0.345	0.200
	11	8	1	2	0.000	0.222	0.085
	12	9	0	1	0.000	1**	0.062
3	1	4	7	4	0.033	0.142	0.119
	2	11	3	5	0.148	0.423	0.287
	3	0	7	1	0.000	1**	0.640
	4	11	2	5	0.423	1	0.430
	5	4	9	6	0.236	0.471	0.284
	6	7	3	5	0.283	0.583	0.370
	7	8	7	6	0.051	0.149	0.101
	8	11	2	2	0.000	0.026	0.020
	9	5	7	5	0.076	0.197	0.077
	10	12	3	6	0.396	0.637	0.407
	11	3	8	5	0.236	0.533	0.421
	12	11	0	1	0.000	1**	0.086

Table 17: Runs Tests for Low-Cost Bidders in LTB Treatments

Session	Subject	[0,50]	(50,100]	Runs	F(r-1)	F(r)	U[F(r-1),F(r)]
1	1	2	9	2	0.000	0.036	0.008
	2	4	3	2	0.000	0.057	0.016
	3	1	7	3	0.250	1.000	0.456
	4	3	6	4	0.107	0.345	0.133
	5	3	3	4	0.300	0.700	0.527
	6	14	3	5	0.101	0.350	0.345
	7	5	10	4	0.005	0.029	0.026
	8	3	4	5	0.543	0.800	0.791
	9	1	5	2	0.000	0.333	0.237
	10	0	3	1	0.000	1**	0.856
	11	3	4	4	0.200	0.543	0.308
	12	7	6	7	0.296	0.500	0.500
2	1	11	1	3	0.167	1.000	0.419
	2	4	4	2	0.000	0.029	0.010
	3	3	12	4	0.033	0.130	0.124
	4	3	3	2	0.000	0.100	0.049
	5	3	4	4	0.200	0.543	0.529
	6	5	2	3	0.095	0.333	0.129
	7	2	4	4	0.400	0.800	0.713
	8	13	0	1	0.000	1**	0.872
	9	7	2	3	0.056	0.250	0.233
	10	7	13	2	0.000	0.000**	0.000
	11	10	3	4	0.045	0.171	0.090
	12	1	2	3	0.667	1.000	0.869
3	1	8	5	5	0.054	0.152	0.109
	2	1	2	2	0.000	0.667	0.479
	3	1	14	3	0.133	1.000	0.416
	4	14	1	2	0.000	0.133	0.111
	5	4	2	4	0.400	0.800	0.425
	6	0	6	1	0.000	1**	0.809
	7	0	2	1	0.000	1**	0.094
	8	1	1	2	0.000	1.000	0.161
	9	5	1	2	0.000	0.333	0.215
	10	1	3	3	0.500	1.000	0.918
	11	19	0	1	0.000	1**	0.809
	12	11	2	3	0.026	0.167	0.133