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Efficient public good provision by lotteries with nonlinear pricing[☆]

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ABSTRACT

In this paper, we introduce nonlinear pricing of lottery tickets to the mechanism of Morgan (2000), in which a lottery is used to finance the public good. In a model with n symmetric agents, we find that incorporating this instrument fully achieves the efficient provision of public good when each agent's initial wealth is sufficiently high. In a model with two asymmetric agents, there exists a nonlinear lottery mechanism that induces efficient public good provision provided that agents are not too heterogenous. Intuitively, the proposed nonlinear pricing rule leads to a decreasing marginal cost for ticket purchase, which provides stronger incentives for agents to make contributions, compared with Morgan (2000).

1. Introduction

It is well known that voluntary contribution leads to the underprovision of public goods because of the free-riding incentive of agents.[1] Morgan (2000) proposes a mechanism that uses lottery revenue to fund public good provision and the lottery prize. This *lottery mechanism* increases public good provision, and thus improves social efficiency. The lottery mechanism provides better incentives, because agents' contributions not only enhance public good provision, but also increase their own chances of winning the lottery prize. However, Morgan's lottery mechanism still underprovides the public good, and the gap between the two (current and efficient) levels asymptotically disappears when the lottery prize approaches infinity.

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[1] For instance, see Bergstrom et al. (1986) and Andreoni (1988).

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In this paper, we propose a revised version of Morgan's (2000) lottery mechanism by incorporating a nonlinear pricing rule for lottery tickets. As in Morgan (2000), we study mechanisms in which the sum of agents' contributions is fully used to finance both the lottery prize and the public good. While agents pay a uniform price for each lottery ticket in Morgan's original (linear pricing) mechanism, in our (nonlinear pricing) mechanism the price of the lottery ticket decreases when one purchases more tickets.[2]

We first analyze a model with n symmetric agents and find that: When agents' (initial) wealth level is sufficiently high, there exists a nonlinear lottery mechanism that induces efficient public good provision in a symmetric pure strategy equilibrium (SPSE), in which both the lottery prize and the public good are financed by agents' total contribution (i.e., their total spending on lottery tickets). This efficient SPSE, however, is not unique—there may exist other equilibria in which agents' total contribution is smaller than the lottery prize, and thus the lottery cannot be held—the public good then is provided under the voluntary contribution mechanism (VCM).[3] Moreover, we show that the efficient SPSE is a unique SPSE given that the lottery is held. This further implies that the designer's commitment on holding the lottery helps in inducing the efficient SPSE, which can be costless for the designer since agent's total contribution is greater than the value of the lottery prize in the efficient SPSE.[4]

In the model with n symmetric agents, we also find that: When agents' wealth level is sufficiently low, the first best cannot be achieved in equilibrium, since the public good is always underprovided. However, we show that, given the existence of SPSE, if the pricing rule becomes more *nonlinear,* a lottery mechanism can increase public good provision and thus induce a higher level of social welfare in equilibrium—because, intuitively, under a more nonlinear pricing rule, the price of lottery tickets decreases at a faster rate when more tickets are purchased, which provides agents stronger incentives to make contributions.[5]

At last, by analyzing a model with two asymmetric agents, we find that: To ensure the existence of a nonlinear lottery mechanism that induces efficient public good provision in equilibrium, two agents cannot be too heterogenous in the sense that the ratio of their marginal utilities from the public good is smaller than 4.68 approximately.

Intuitively, in the lottery mechanism of Morgan (2000), when an agent increases her contribution (on lottery tickets), there are two effects: First, it increases the level of the public good for all agents, which is referred to as the "positive externality effect"; second, it decreases all other agents' chances of winning the lottery prize, which is referred to as the "negative externality effect." The positive externality effect causes the free-ride problem, because when an arbitrary agent makes her contribution decision, the positive externality (to other agents) on public good provision is ignored, which tends to lower the contribution level (relative to the first best); while the negative externality effect alleviates the free-ride problem due to one's ignorance of the negative externality (to other agents) on lottery winning, which tends to raise the contribution level. In summary, in the lottery mechanism of Morgan (2000), the competition for the lottery prize introduces a negative externality effect, which can partially offset the influence of the positive externality effect. When the lottery prize becomes larger, the negative externality effect becomes larger, and thus more positive externality effect is offset—however, in Morgan's mechanism, the negative externality effect can never completely offset the positive externality effect—i.e., the free-ride problem can be alleviated, but not eliminated completely.

In our non-linear pricing (resp. Morgan's linear pricing) lottery mechanism, the unit price of lottery tickets decreases (resp. stays constant) when more tickets are purchased. Thus, the adoption of the non-linear pricing rule, which leads to a decreasing marginal cost when one buys more lottery tickets, provides stronger motivations for agents than the lottery mechanism of Morgan (2000). Intuitively, the nonlinear pricing rule exaggerates the negative externality effect, and thus further offsets the positive externality effect. This explains why efficient public good provision can be achieved in our nonlinear pricing lottery mechanisms under some conditions.

Solutions had been offered to resolve the free-rider problem on public good provision before (Morgan, 2000),[6] which include the mechanisms proposed by Groves and Ledyard (1977) and Walker (1981), and voluntary mechanisms with "provision points" (Bagnoli and Lipman, 1989; Admati and Perry, 1991). These mechanisms differ in their informational assumptions from the lotteries of Morgan (2000) and this paper. For instance, in the mechanisms of Groves and Ledyard (1977) and Walker (1981), the government solicits information from the consumers about their preferences for public goods, which is used to determine its purchases of public goods and the taxes levied on consumers to finance those purchases. In contrast, in our lottery mechanisms, the designer—which can be a (local) government or other organizations with limited abilities to impose taxes (e.g., civic associations and charities)—has the information on agents' preferences for public goods, but has no coercive power to impose taxes or penalties on agents. In addition, experimental evidence for these mechanisms is some-

[2] In both (Morgan, 2000) and this paper, the sum of agents' contributions is fully used to finance both the lottery prize and the public good.

[3] By the rule of a lottery mechanism of this paper and (Morgan, 2000), if the agents' total contribution is smaller than the lottery prize, the lottery will not be held, the same amount of money collected will be returned to each agent, and the public good is provided by the voluntary contribution mechanism (VCM).

[4] We also point out that an agent's equilibrium payoff is strictly higher in the efficient SPSE, and thus ex-ante communications between agents can help them to coordinate on the efficient SPSE.

[5] Thus, in our lottery mechanisms, the social planner needs to know an agent's wealth level when choosing the mechanism that maximizes the social welfare. In contrast, the social planner does not have to know the agents' wealth level in some conventional mechanisms (see, for instance, Hurwicz et al., 1995).

[6] Chowdhury et al. (2013) analyze a best-shot group contest, in which groups compete to win a group-specific public good prize, and show that there is severe free-riding in any equilibrium.

what mixed (Harstad and Marrese, 1982; Ledyard, 1995; Chen and Tang, 1998). Morgan and Sefton (2000) experimentally show that public good provision is higher when financed by lottery proceeds (than voluntary contributions), and lotteries with large prizes are more effective. Consistently, field experiments by Landry et al. (2006) and Lange et al. (2007) show that a lottery mechanism is superior than voluntary contributions for fundraising. A few laboratory experiments compare the performance of lottery and all-pay auction as fund-raising mechanisms (Orzen, 2008; Schram and Onderstal, 2009; Corazzini et al., 2010), and the results are contingent on various parameters. For example, with heterogenous income and incomplete information about income levels, Corazzini et al. (2010) show that lottery is better than all-pay auction.

Furusawa and Konishi (2011) study public good provision with voluntary participation in a quasilinear economy by proposing a "free-riding-proof core" that has support from both cooperative and noncooperative games. This free-riding-proof core endogenously determines a contribution group, public good provision level, and how to share the provision costs. Kolmar and Wagener (2012) study a model in which the lottery prize is fully financed by a lump-sum tax to each agent, and one's probability of winning depends on agents' contributions to the public good. Giebe and Schweinzer (2014) analyze a model in which both the lottery prize and the public good are financed by consumption taxes on private goods, and when an agent increases her private good consumption, both the value of the lottery prize and her chance of winning it increase.[7] Our paper differ from those studies because our lottery mechanism does not rely on any coercive power (e.g., a coercive tax) to finance the lottery prize or/and the public good, whereby agents make their contributions voluntarily, as in Morgan (2000). The "voluntariness" in this paper and Morgan (2000) means that the agents in the lottery mechanisms have freedom not to participate in the lottery by purchasing zero tickets—i.e., an agent is allowed to make zero contribution and can still obtain the benefit of a public good that is provided by other agents.[8] It should be further pointed out that in a tax lottery scheme of Giebe and Schweinzer (2014), two fixed proportions of the total tax revenue on private good consumption are used to provide the public good and the lottery prize, respectively. Thus, besides our lottery mechanism not relying on a coercive power of taxation, another important difference between this paper and Giebe and Schweinzer (2014) lies in that the value of the lottery prize is exogenously given in our mechanism, and is endogenously determined by the agents' private good consumption level in theirs.

Kolmar and Sisak (2014) analyze a model with asymmetric agents, where a multi-prize contest can be used to improve public good provision, and offer conditions that guarantee the efficiency. Franke and Leininger (2014) study a setting with two asymmetric agents, and propose to use a "biased" unfair lottery for efficient public good provision.[9] Franke and Leininger (2018) further show that biased lotteries can implement Lindahl pricing of efficient public good provision in a Nash equilibrium.[10] Different from the above studies, we analyze two models with n symmetric players and two asymmetric players, respectively, in which only fair lotteries (i.e., lotteries without identity discrimination) are considered throughout this paper, such as in Morgan (2000).

2. The model

There are $n \geq 2$ agents, who derive utility from consuming a numeraire private good g and a public good G. Every agent i has an initial wealth budget of $w_i = w > 0$. In the first part of this section (Section 2.1), we assume that w is sufficiently high such that an interior optimum exists in a model with n symmetric agents; in the second part of this section (Section 2.2), we study a contrasting case in which agents' wealth constraints are binding at equilibrium.[11]

Each agent i has a quasilinear utility function, which is given by

$$u_i(g_i, G) = g_i + h(G), \tag{1}$$

where g_i stands for agent i's consumption of the numeraire (private) good and G stands for the level of public good provided.[12] Function $h(\cdot)$ measures how the agents value the public good. We assume that agents experience diminishing marginal utility from the provision of the public good, i.e., $h'(\cdot) > 0$ and $h''(\cdot) < 0$. In addition, without loss of generality, we assume that $h'(0) > 1/n > h'(\infty)$; later we find that this condition ensures that the efficient level of public good is strictly positive and finite.

[7] Falkinger (1994) proposes an incentive scheme in which each agent gets a reward or pays a penalty, depending on the deviation of their contribution from the mean contribution. Bierbrauer (2009) extends the model of optimal income taxation of Mirrlees (1971) by using a mechanism design approach. In these mechanisms, a coercive power (e.g., a reward/penalty or taxation) is required.

[8] Before Saijo and Yamato (1999), various mechanisms (for instance, Groves and Ledyard, 1977, and a number of following studies) are proposed to implement the Lindahl correspondence in Nash equilibria, in which agents must participate in the mechanism proposed—this neglects non-excludability of public goods. Saijo and Yamato (1999) study a- two-stage mechanism where agents make participation decisions in the first stage, and the participating agents choose their strategies in the second stage, with other agents' participation decisions being public information. They show that participation of all agents is not an equilibrium in many situations, and at equilibrium the set of economies for which every agent chooses participation becomes smaller and vanishes as the number of agents grows large.

[9] Duffy and Matros (2012) study a similar exercise in an all-pay auction setting.

[10] In addition, Franke and Nandeibam (2021) propose a probabilistic fine scheme in a model of public bad, where one's probability of being fined takes a variant form of lotteries. They show that there exists a fine scheme that achieves the first best in a pure strategy equilibrium.

[11] In addition, in the third part of this section (Section 2.3), we look at a model with two (i.e., $n = 2$) asymmetric agents; moreover, in the fourth part of this section (Section 2.4), we analyze a lottery mechanism with a nonlinear pricing rule which is more general than that studied previously.

[12] We retain the assumptions of Morgan (2000) on quasilinear utility and numeraire (private) good.

Social Optimum

Consider the first-best case in which a designer chooses the level of public good G to maximize the social welfare W, where $W = \sum_{i=1}^{n} (g_i + h(G))$, subject to the following social budget constraint $\sum_{i=1}^{n} w_i = G + \sum_{i=1}^{n} g_i$. Using the above expressions of W and social budget constraint, we derive that $W = \sum_{i=1}^{n} w_i - G + n \times h(G)$, which is a function of a single variable G. It is straightforward to obtain that at an interior optimum, the efficient level of public good provision G^* solves[13]

$$h'(G^*) = \frac{1}{n}, \tag{2}$$

which is the well-known Samuelson criterion for welfare maximization.

Voluntary Contributions

Consider the case in which the designer relies on voluntary contributions for provision of the public good. Let z_i denote the amount of wealth contributed by agent i. We focus on a symmetric pure strategy equilibrium (SPSE) in which each agent chooses to contribute $z^V \in [0, w]$, and thus the total amount of public good provided in equilibrium is $G^V = nz^V$. Bergstrom et al. (1986) show that the level of public good under voluntary contributions, G^V, solves[14]

$$h'(G^V) = 1. \tag{3}$$

Comparing (2) and (3), the following result is obtained.

Proposition 1 (Bergstrom et al.(1986)). *Voluntary contribution underprovides the public good relative to the first-best level—i.e., $G^V < G^*$.*

Intuitively, when agent i increases her contribution for pubic good provision, it has a positive externality for other agents. However, when deciding on the level of z_i, agent i completely ignores the positive externality for all other agents because she only focuses on maximizing her own expected payoff. Thus, each agent tends to under-contribute relative to the first-best level.

2.1. Lottery mechanism with nonlinear pricing

We introduce nonlinear pricing to the lottery mechanism of Morgan (2000). All entrants simultaneously choose the number of tickets they want to purchase. Agent i needs to pay $c(x_i) = x_i^{\alpha}$ to buy x_i tickets, where $\alpha \in (0, 1]$, which is a nonlinear pricing (resp. linear pricing) rule for $\alpha \in (0, 1)$ (resp. $\alpha = 1$). Ticket revenue $\sum_{i=1}^{n} x_i^{\alpha}$ is collected to finance both the public good and the lottery prize.[15] A lottery with prize value R, where $R \geq 0$, will be held if and only if $\sum_{j=1}^{n} x_j^{\alpha} \geq R$, which means that the total revenue from ticket sales is (weakly) larger than the prize value.

When the lottery is held, the revenue from ticket sales net of the budget for the prize will be used as public good provision—i.e., $G = \sum_{j=1}^{n} x_j^{\alpha} - R$. In this case, the probability that agent i wins the prize is given by

$$p_i(x_i, x_{-i}) = \frac{x_i}{\sum_{j=1}^{n} x_j},$$

where x_i is the number of tickets purchased by agent i and x_{-i} denotes the number of tickets purchased by the other agents. Otherwise, if $\sum_{j=1}^{n} x_j^{\alpha} < R$, which means the agents' total contribution on lottery purchase is strictly smaller than the value of the lottery prize, the lottery will not be held, the same amount of money collected will be returned to each agent, and the public good is provided by voluntary contributions.

Our main result is as follows.

Theorem 1. *When $w \geq \left(\frac{1}{1-\alpha}\right)\frac{G^*}{n}$, a nonlinear pricing lottery mechanism with (α, R) such that $\alpha \in [\frac{n-1}{n}, 1)$ and $R = \frac{\alpha}{1-\alpha} G^*$ induces efficient public good provision in a symmetric pure strategy equilibrium (SPSE), in which each agent pays $x^{*\alpha}$ to buy x^* tickets, where $x^* = \left(\frac{R}{\alpha n}\right)^{\frac{1}{\alpha}}$.*

[13] To see this, the social planner chooses G to maximize the social welfare W. Using the above expressions of social welfare and social budget constraint, it can be derived that $W = \sum_{i=1}^{n} w_i - G + n \times h(G)$, and its FOC implies that $h'(G^*) = 1/n$.

[14] To see this, given the contribution of every other agent being the equilibrium amount z^V, agent i chooses $z_i \in [0, w]$ to maximize her expected utility $u_i(z_i) = w_i - z_i + h(z_i + (n-1)z^V)$, with FOC being that $h'(z_i + (n-1)z^V) - 1 = 0$. At equilibrium, it is optimal for player i to choose $z_i = z^V$, which, together with the above FOC, leads to $h'(G^V) = 1$.

[15] Actually, our main results do not rely on this particular form of the payment function $c(x) = x^{\alpha}$ (i.e., one needs to pay x^{α} for x tickets), in the sense that any other strictly concave, continuous payment function, with $c(0) = 0$, $c'(x) > 0$ and $c''(x) < 0$, could yield similar results—see Subsection 2.4 for a detailed analysis with a more general nonlinear pricing rule. We focus on the case of $c(x) = x^{\alpha}$ for tractability and brevity: It guarantees the existence of an SPSE without any additional conditions on the payment function; it also allows us to use a single variable α to measure the extent of the nonlinearity.

Proof. We seek to show that $x^* = \left(\frac{R}{\alpha n}\right)^{\frac{1}{\alpha}}$ is the equilibrium.[16] The following is a road map for the proof. In Step 1, we fix all others' ticket numbers at x^*, and show that a representative agent i would also choose x^* which dominates any x_i such that $x_i^\alpha + (n-1)(x^*)^\alpha \geq R$ (i.e., when the lottery is successful). In Step 2, we further show that from agent i's point of view, choosing x^* dominates any x_i such that $x_i^\alpha + (n-1)(x^*)^\alpha < R$ (i.e., when the lottery fails).

We first look at Step 1. Let $\underline{x} = [R-(n-1)(x^*)^\alpha]^{\frac{1}{\alpha}}$, which implies that $x_i^\alpha + (n-1)(x^*)^\alpha \geq R$ when $x_i \in [\underline{x}, w^{\frac{1}{\alpha}})$. Agent i's problem is given by

$$\max_{x_i \in [\underline{x}, w^{\frac{1}{\alpha}})} u_i(x_i, x^*) = w + \frac{x_i}{(n-1)x^* + x_i} R - x_i^\alpha + h\Big((n-1)(x^*)^\alpha + x_i^\alpha - R\Big). \tag{4}$$

Let $e^* = (x^*)^\alpha = \frac{R}{\alpha n}$, $e_i = x_i^\alpha$, and $r = \frac{1}{\alpha} \in (1, \frac{n}{n-1}]$. Also define $\underline{e} = [R-(n-1)e^*]$, which implies that $e_i + (n-1)e^* \geq R$ when $e_i \in [\underline{e}, w)$. The above problem can be rewritten as[17]

$$\max_{e_i \in [\underline{e}, w)} u_i(e_i, e^*) = w + \frac{e_i^r}{e_i^r + (n-1)(e^*)^r} R - e_i + h((n-1)e^* + e_i - R). \tag{5}$$

We next verify that $e_i = e^*$ satisfies the first-order condition:

$$\frac{\partial u_i}{\partial e_i} = \frac{(n-1)(e^*)^r e_i^{r-1} rR}{\left[e_i^r + (n-1)(e^*)^r\right]^2} - 1 + h'((n-1)e^* + e_i - R) = 0. \tag{6}$$

Note that using $e^* = \frac{R}{\alpha n}$ and $^*R = \frac{\alpha}{1-\alpha} G^*$, we can derive that

$$h'((n-1)e^* + e_i - R)|_{e_i = e^*} = h'(ne^* - R) = h'(\tfrac{1-\alpha}{\alpha} R) = h'(G^*). \tag{7}$$

Using (7) and (2), (6) can be rewritten as

$$\frac{\partial u_i}{\partial e_i}|_{e_i = e^*} = \frac{(n-1)rR}{n^2 e^*} - \left(\frac{n-1}{n}\right) = 0. \tag{8}$$

To facilitate our analysis, consider the following Tullock contest model: There are n agents who exert effort to compete for a prize R in a Tullock contest with a success function of

$$\tilde{p}_i(e_i, e_{-i}) = \frac{e_i^r}{e_i^r + \sum_{j \neq i}^r e_j^r}, \tag{9}$$

where $r \in (1, \frac{n}{n-1}]$, and each agent's marginal cost of exerting effort is $\frac{n-1}{n}$. From Szidarovszky and Okuguchi (1997) and Schweinzer and Segev (2012), a unique symmetric pure-strategy equilibrium $\tilde{e}^*$ exists, where

$$\tilde{e}^* = \left(\frac{n}{n-1}\right) \frac{(n-1)r}{n^2} R = \frac{R}{\alpha n} = e^*. \tag{10}$$

It follows that agent i's expected payoff

$$\tilde{u}_i(e_i, e^*) = \frac{e_i^r}{e_i^r + (n-1)(e^*)^r} R - \left(\frac{n-1}{n}\right) e_i \tag{11}$$

is maximized at $e_i = e^*$ when others' effort is e^*.

Now we are ready to show that in our original model, $u_i(e_i, e^*)$ is also maximized at $e_i = e^*$. For this purpose, first we seek to show the following relation between $\tilde{u}_i(e_i, e^*)$ and $u_i(e_i, e^*)$:

$$\begin{aligned} u_i(e_i, e^*) &< \left(w + h(G^*) - \frac{e^*}{n}\right) + \tilde{u}_i(e_i, e^*), \text{ if } e_i \neq e^*; \\ u_i(e_i, e^*) &= \left(w + h(G^*) - \frac{e^*}{n}\right) + \tilde{u}_i(e_i, e^*), \text{ if } e_i = e^*. \end{aligned} \tag{12}$$

This relation holds because the straight line that is tangent to concave function $h((n-1)e^* + e_i - R)$ at $e_i = e^*$ is

$$h(G^*) + \frac{1}{n}(e_i - e^*).$$

[16] Recall that in the main text, we have shown that with (15), the corresponding level of public good provision is efficient, as $n(x^*)^\alpha - R = G^*$; each agent's equilibrium spending on lottery tickets is not greater than her initial wealth level and the proposed lottery prize value is no greater than the sum of all agents' wealth.

[17] Note that showing that $x_i = x^*$ is a global maximum of $u_i(x_i, x^*)$ is equivalent to showing that $e_i = e^*$ is a global maximum of $u_i(e_i, e^*)$. We focus on the latter in the following analysis, because it is a standard maximization problem in a Tullock contest setting and thus we can borrow some previous results from that literature—Schweinzer and Segev (2012) show that a symmetric pure strategy equilibrium (SPSE) exists for $r \in (0, \frac{n}{n-1}]$, and note that $\frac{n}{n-1} > 1$ indicates that such a SPSE exists for $r \in (1, \frac{n}{n-1}]$—which will significantly shorten our proof compared with showing the former directly.

Note that when $e_i = e^*$,

$$h'((n-1)e^* + e_i - R) = h'(G^*) = \frac{1}{n}.$$

From the concavity of $h(\cdot)$, we have

$$h(G^*) + \frac{1}{n}(e_i - e^*) > h((n-1)e^* + e_i - R),\ \text{if } e_i \neq e^*;$$

$$h(G^*) + \frac{1}{n}(e_i - e^*) = h((n-1)e^* + e_i - R),\ \text{if } e_i = e^*.$$

Replacing $h((n-1)e^* + e_i - R)$ with $h(G^*) + \frac{1}{n}(e_i - e^*)$ in (5), we obtain the bound $\left(w + h(G^*) - \frac{1}{n}e^*\right) + \tilde{u}_i(e_i, e^*)$ in (12):

$$\begin{aligned} u_i(e_i, e^*) &= w + \frac{e_i^r}{e_i^r + (n-1)(e^*)^r}R - e_i + h((n-1)e^* + e_i - R) \\ &\leq w + \frac{e_i^r}{e_i^r + (n-1)(e^*)^r}R - e_i + h(G^*) + \frac{1}{n}(e_i - e^*) \\ &= \left(w + h(G^*) - \frac{e^*}{n}\right) + \tilde{u}_i(e_i, e^*). \end{aligned}$$

Because (12) holds and $\tilde{u}_i(e^*, e^*) \geq \tilde{u}_i(e_i, e^*)$, we must have $u_i(e^*, e^*) \geq u_i(e_i, e^*)$, which means that e^* is indeed the optimal choice of agent i among all e_i. This further implies that x^* is indeed the optimal choice of agent i among all x_i such that the lottery is held successfully.

We now turn to Step 2 and show that for agent i, choosing x^* dominates any x_i such that $(x_i)^\alpha + (n-1)(x^*)^\alpha < R$, i.e., when $x_i < \underline{x}$ and the lottery fails. Note that the lottery can fail if $(n-1)(x^*)^\alpha = (n-1)\frac{R}{\alpha n} < R$, i.e., if $\alpha > \frac{n-1}{n}$. When $(x_i)^\alpha + (n-1)(x^*)^\alpha < R$, the voluntary contribution game is triggered. Each agent gets a utility of

$$u^V = w + h(G^V) - z, \tag{13}$$

where $G^V = nz$, which is defined in (3), and z is each agent's equilibrium contribution.

Next we show that $u_i(e^*, e^*) > u^V$. Using $e^* = \frac{R}{\alpha n}$ and $R = \frac{\alpha}{1-\alpha}G^*$, we obtain that

$$u_i(e^*, e^*) = w + \frac{R}{n} - \frac{R}{\alpha n} + h(G^*) = w - \frac{1}{n}G^* + h(G^*). \tag{14}$$

Using (13), (14), and $z = \frac{G^V}{n}$, we get $u_i(e^*, e^*) > u^V$ as

$$w + h(G^*) - \frac{G^*}{n} \geq w + h(G^V) - \frac{G^V}{n}.$$

The above inequality holds since $h'(G) > 1/n$ for $0 \leq G < G^*$, which implies that $h(G) - G/n$ increases in G. □

In the proof, we show that

$$x^* = \left(\frac{R}{\alpha n}\right)^{\frac{1}{\alpha}} \tag{15}$$

is the equilibrium. The proof consists of two steps: In Step 1, we fix all others' ticket numbers at x^*, and show that a representative agent i would also choose x^* which dominates any x_i such that ${x_i}^\alpha + (n-1)(x^*)^\alpha \geq R$ (i.e., when the lottery is held). In Step 2, we show that choosing x^* dominates any x_i such that ${x_i}^\alpha + (n-1)(x^*)^\alpha < R$ (i.e., when the lottery fails).

Three points about the result of Theorem 1 are emphasized as follows: First, note that $n(x^*)^\alpha = \frac{R}{\alpha} > R$ as $\alpha < 1$, so with this level of contribution, the lottery is held. Second, it can be shown that with (15), the corresponding level of public good provision is

$$n(x^*)^\alpha - R = G^*, \tag{16}$$

which is efficient. Third, in the proposed lottery mechanism with $\alpha \in [\frac{n-1}{n}, 1)$ and $R = \frac{\alpha}{1-\alpha}G^*$: ($i$) each agent's equilibrium spending on lottery tickets is not greater than her initial wealth level and (ii) the proposed lottery prize value is not greater than the sum of all agents' wealth.[18] To see this, when $w \geq \left(\frac{1}{1-\alpha}\right)\frac{G^*}{n}$, we have

$$(x^*)^\alpha = \frac{R}{\alpha n} = \left(\frac{1}{1-\alpha}\right)\frac{G^*}{n} \leq w;$$

both $\alpha \in [\frac{n-1}{n}, 1)$ and $w \geq \left(\frac{1}{1-\alpha}\right)\frac{G^*}{n}$ imply that

$$R = \left(\frac{\alpha}{1-\alpha}\right)G^* \leq \left(\frac{1}{1-\alpha}\right)G^* \leq nw.$$

[18] Given an $\alpha \in [\frac{n-1}{n}, 1)$, overprovision (resp. underprovision) of the public good occurs if a larger (resp. smaller) lottery prize R' is adopted, where $R' > \frac{\alpha}{1-\alpha}G^*$ (resp. $R' < \frac{\alpha}{1-\alpha}G^*$), provided that the above (i) and (ii) hold.

Next, we discuss the possibility of other equilibria under the lottery mechanisms proposed in this paper. For simplicity and illustrative purpose, in following analysis with n symmetric agents, we focus on symmetric pure strategy equilibria (SPSE). We show that the above efficient SPSE is not unique. To see this, consider a non-linear pricing lottery mechanism of Theorem 1, with $\alpha \in [\frac{n-1}{n}, 1)$ and $R = \frac{\alpha}{1-\alpha}G^*$. It can be shown that, besides the efficient SPSE characterized in Theorem 1, there may exist other SPSE, in which agents' total contribution is sufficiently small in the sense that the lottery cannot be held—and then the public good is provided under the voluntary contribution mechanism (VCM).

For instance, in the following analysis we show that there exists a "zero-contribution-no-lottery SPSE," in which each agent initially makes zero contribution and thus the lottery cannot be held—the public good is then provided under the VCM. It is straightforward to see that why every agent bidding zero constitutes an equilibrium: Given that all other agents make zero contribution—i.e., each agent buys zero lottery tickets, it is optimal for agent i to choose $x_i = 0$. For agent i, choosing $x_i = 0$ dominates any other $x_i > 0$ due to the following reasons: On the one hand, choosing a sufficiently small x_i such that $x_i \in (0, R^{\frac{1}{\alpha}})$ cannot strictly increase her payoff (compared with that under VCM) since the lottery cannot be held anyway with $x_i^{\alpha} < R$;[19] on the other hand, choosing a sufficiently large x_i such that $x_i \in [R^{\frac{1}{\alpha}}, w]$ cannot increase her payoff (compared with that under VCM) either, because in this case agent i is the only one who contributes to the public good, despite that she wins the lottery prize R with probability one.[20]

Notice that from an arbitrary agent's perspective, she would strictly prefer an efficient SPSE of Theorem 1 to any other SPSE in which the pubic good is underprovided, since an efficient public good provision is achieved in the former, which maximizes the sum of all agents' expected payoffs. Thus, an ex-ante communication among agents (before they purchasing lottery tickets) can help them to coordinate on an efficient SPSE, in which the public good is provided efficiently and thus each agent is better off than that with the public good being underprovided.

Moreover, the following proposition shows that, despite of the existence of other SPSE where the lottery fails, the efficient SPSE of Theorem 1 is a unique SPSE provided that the lottery is held.

Proposition 2. *When $w \geq \left(\frac{1}{1-\alpha}\right)\frac{G^*}{n}$, in a nonlinear pricing lottery mechanism in which $\alpha \in [\frac{n-1}{n}, 1)$ and $R = \frac{\alpha}{1-\alpha}G^*$, the symmetric pure strategy equilibrium (SPSE) characterized in Theorem 1 is a unique SPSE, provided that the lottery is held.*

Proof. We seek to show that: Any other SPSE candidate with each agent's lottery purchase amount being x^s is an equilibrium if and only if $x^s = x^*$, i.e., any other x^s, such that either $x^s < x^*$ or $x^s > x^*$, cannot be an equilibrium.

From the proof of Theorem 1, we can see that to ensure that each agent buying x^s constitutes an equilibrium, it should be optimal for agent i to buy x^s tickets given every one else purchasing x^s tickets. In this case, from the first order condition of agent i, we derive the following necessary condition for the existence of an SPSE:

$$\frac{\partial u_i}{\partial e_i}|_{e_i=e^s} = \frac{(n-1)rR}{n^2 e^s} - \Big(1 - h'(ne^s - R)\Big) = 0,$$

where $e_i = x_i^{\alpha}$, $e^s = (x^s)^{\alpha}$, and $r = \frac{1}{\alpha} \in (1, \frac{n}{n-1}]$. The above equality holds at $x^s = x^*$—i.e., at $e^s = (x^*)^{\alpha} = e^*$—to see this, using $e^* = \frac{R}{\alpha n}$, $r = \frac{1}{\alpha}$, $ne^* - R = G^*$ and $h'(G^*) = \frac{1}{n}$, we obtain that

$$\begin{aligned}\frac{\partial u_i}{\partial e_i}|_{e_i=e^*} &= \frac{(n-1)rR}{n^2 e^*} - \Big(1 - h'(ne^* - R)\Big) \\ &= \frac{(n-1)R}{n^2\alpha}\left(\frac{\alpha n}{R}\right) - \Big(1 - h'(G^*)\Big) \\ &= \frac{(n-1)}{n} - \left(1 - \frac{1}{n}\right) = 0.\end{aligned}$$

When $x^s < x^*$, we have $\frac{\partial u_i}{\partial e_i}|_{e_i=e^s} = \frac{(n-1)rR}{n^2 e^s} - \Big(1 - h'(ne^s - R)\Big) > 0$ since, compared with $\frac{\partial u_i}{\partial e_i}|_{e_i=e^*} = \frac{(n-1)rR}{n^2 e^*} - \Big(1 - h'(ne^* - R)\Big) = 0$, the first term $\frac{(n-1)rR}{n^2 e^s}$ gets larger and second term $\Big(1 - h'(ne^s - R)\Big)$ gets smaller. While when $x^s > x^*$, we have $\frac{\partial u_i}{\partial e_i}|_{e_i=e^s} = \frac{(n-1)rR}{n^2 e^s} - \Big(1 - h'(ne^s - R)\Big) < 0$ since, compared with $\frac{\partial u_i}{\partial e_i}|_{e_i=e^*} = \frac{(n-1)rR}{n^2 e^*} - \Big(1 - h'(ne^* - R)\Big) = 0$, the first term $\frac{(n-1)rR}{n^2 e^s}$ gets smaller and second term $\Big(1 - h'(ne^s - R)\Big)$ gets larger. In other words, the necessary condition can never hold for any $x^s < x^*$ or $x^s > x^*$. Therefore, given that the lottery holds, every one buying x^s tickets is an equilibrium if and only if $x^s = x^*$, so the efficient SPSE of Theorem 1 is a unique SPSE. □

The above result indicates that in a lottery mechanism of Theorem 1, there is a way to avoid an equilibrium in which the lottery cannot be held: Ex ante, the designer, such as the government, can announce that the lottery will be held for certain by committing that if the total contribution of agents is less than the prize value required, she will fully cover the difference. Agents will trust this commitment when the designer has an commitment power—for instance, she has some property as collateral, or she can first borrow R from somewhere else—note that this will not be an issue for the designer,

[19] Here, we focus on the case in which $w \geq R^{\frac{1}{\alpha}}$. It is clear to see that agent i has no incentive to deviate from $x_i = 0$ when $w < R^{\frac{1}{\alpha}}$.

[20] Besides the zero-effort-no-lottery SPSE, there can be other equilibria, in which agents' total contribution is sufficiently small in the sense that the lottery fails and the pubic good is then provided under the VCM.

because when the lottery (with prize $R = \frac{\alpha}{1-\alpha}G^*$) holds for certain, there is a unique SPSE in which both the lottery prize (R) and the public good (G^*) are fully covered by the total contribution of agents ex post.

2.2. When the wealth constraint binds

In the previous analysis, an agent's initial wealth level w is assumed to be sufficiently high such that an interior optimum always exists when choosing the number of lottery tickets she purchases. In this subsection, we consider a contrasting case in which agents' wealth constraints are binding in equilibrium—i.e., each agent has a corner solution rather than an interior solution when maximizing her utility.

In Theorem 1, we have shown that when $w \geq \left(\frac{1}{1-\alpha}\right)\frac{G^*}{n}$, a lottery mechanism with $\alpha \in [\frac{n-1}{n}, 1)$ and $R = \frac{\alpha}{1-\alpha}G^*$ induces efficient public good provision in equilibrium, under which each agent spends exactly $\frac{R}{\alpha n} > 0$ on lottery tickets and $w - \frac{R}{\alpha n} \geq 0$ on private good consumption. Notice that when $w = \left(\frac{1}{1-\alpha}\right)\frac{G^*}{n}$, each agent spends all her wealth w on lottery tickets and zero on private good consumption, since

$$\frac{R}{\alpha n} = \frac{1}{\alpha n}\left(\frac{\alpha}{1-\alpha}G^*\right) = \left(\frac{1}{1-\alpha}\right)\frac{G^*}{n} = w.$$

In contrast, when $w < \left(\frac{1}{1-\alpha}\right)\frac{G^*}{n}$ and all else remains unchanged, it can be shown that each agent has an incentive to spend all her wealth on lottery tickets in equilibrium—i.e., $(x^*)^\alpha = w$. However, the level of public good G is strictly lower than its efficient level G^*, because

$$G = nw - R < \left(\frac{1}{1-\alpha}\right)G^* - \left(\frac{\alpha}{1-\alpha}\right)G^* = G^*.$$

Thus, a lottery mechanism with $\alpha \in [\frac{n-1}{n}, 1)$ and $R = \frac{\alpha}{1-\alpha}G^*$, which induces efficient public good provision when $w \geq \left(\frac{1}{1-\alpha}\right)\frac{G^*}{n}$, cannot achieve the first best any longer when $w < \left(\frac{1}{1-\alpha}\right)\frac{G^*}{n}$.

In the following analysis, we focus on lottery mechanisms with $\alpha \in [\frac{n-1}{n}, 1)$ and $R \in (0, nw - G^V]$, under which an SPSE exists and $n(x^*)^\alpha \geq R$, which means that the proposed lottery is held in equilibrium.[21]

Next, in Lemmas 1 and 2, we show that with the wealth constraint $w < \left(\frac{1}{1-\alpha}\right)\frac{G^*}{n}$ and given $\alpha \in [\frac{n-1}{n}, 1)$, there are two types of equilibria when the value of R varies: First, when $R < \overline{R}(\alpha, w)$, where $\overline{R}(\alpha, w)$ is determined by

$$\left(\frac{n-1}{\alpha n}\right)\overline{R}(\alpha, w) = nw\Big(1 - h'(nw - \overline{R}(\alpha, w))\Big), \tag{17}$$

and it can be further shown that $\overline{R}(\alpha, w) \in (0, nw - G^V)$,[22] and when $R < \overline{R}(\alpha, w)$, each agent's wealth constraint does not bind in equilibrium—i.e., $(x^*)^\alpha < w$. Second, when $R \geq \overline{R}(\alpha, w)$, each agent's wealth constraint binds in equilibrium, i.e., $(x^*)^\alpha = w$.

Lemma 1. *For any lottery mechanism with (α, R) such that $\alpha \in [\frac{n-1}{n}, 1)$ and $R \in (0, \overline{R}(\alpha, w)]$, there exists an SPSE under which each agent spends $(x^*)^\alpha$ to buy x^* lottery tickets, where x^* is determined by*

$$\left(\frac{n-1}{\alpha n}\right)R = n(x^*)^\alpha\Big(1 - h'(n(x^*)^\alpha - R)\Big). \tag{18}$$

Both an agent's spending on lottery tickets (henceforth: individual spending) and the level of public good provision, i.e., both $(x^*)^\alpha$ and $n(x^*)^\alpha - R$, increase in R. More specifically, when R increases from 0 to $\overline{R}(\alpha, w)$, individual spending $(x^*)^\alpha$ increases from $\frac{1}{n}G^V$ to w and public good $n(x^*)^\alpha - R$ increases from G^V to $nw - \overline{R}(\alpha, w)$, respectively.

Proof. See Appendix. □

Lemma 2. *For a given lottery mechanism denoted by (α, R) such that $\alpha \in [\frac{n-1}{n}, 1)$ and $R \in (\overline{R}(\alpha, w), nw - G^V]$, there exists an SPSE under which each agent spends all her wealth on lottery tickets—i.e., $(x^*)^\alpha = w$ and thus the level of public good provision, $n(x^*)^\alpha - R = nw - R$, decreases in R. More specifically, when R increases from $\overline{R}(\alpha, w)$ to $nw - G^V$, individual spending stays unchanged as $(x^*)^\alpha = w$, while public good $n(x^*)^\alpha - R$ decreases from $nw - \overline{R}(\alpha, w)$ to G^V.*

Proof. See Appendix. □

By Theorem 1, there always exists a nonlinear pricing lottery mechanism with $\alpha \in [\frac{n-1}{n}, 1)$ and $R = \frac{\alpha}{1-\alpha}G^*$, which induces efficient public good provision in an SPSE provided that $w \geq \left(\frac{1}{1-\alpha}\right)\frac{G^*}{n}$. Notice that the RHS of the above equality (i.e., $\left(\frac{1}{1-\alpha}\right)\frac{G^*}{n}$) is strictly increasing in α, which implies that to ensure the existence of such an efficient SPSE, each agent's individual wealth w must be no less than $\left(\frac{1}{1-\alpha}\right)\frac{G^*}{n}\big|_{\alpha=\frac{n-1}{n}} = G^*$. In other words, when this condition does not hold—i.e., when

[21] We do not consider the case in which $R > nw - G^V$, because when it holds, it is either the case that a pure strategy equilibrium doesn't exist, or it induces a lower level of the public good in equilibrium than that under voluntary contribution, as $nw - R < nw - (nw - G^V) = G^V$.

[22] To see that $\overline{R}(\alpha, w) \in (0, nw - G^V)$ is true, note that when $\overline{R}(\alpha, w)$ increases from zero, the LHS of (17) increases with $\overline{R}(\alpha, w)$ from zero—but the RHS decreases with $\overline{R}(\alpha, w)$ from strictly positive, and it would be zero if $\overline{R}(\alpha, w) = nw - G^V$ as $RHS = nw(1 - h'(G^V)) = 0$. Thus, single crossing implies that the unique solution must be strictly in between zero and $nw - G^V$.

$w < G^*$, there exists no lottery mechanism of Theorem 1 in which efficient public good provision is induced in an SPSE. We summarize the result obtained in the case with $w < G^*$ in the following theorem.

Theorem 2. *With individual wealth constraint $w < G^*$,*[23] *of all lottery mechanisms with (α, R) such that $\alpha \in [\frac{n-1}{n}, 1)$ and $R \in (0, nw - G^V]$, in which an SPSE exists for every pair of α and R, social welfare is maximized in the lottery mechanism with $\alpha = \frac{n-1}{n}$ and $R = \overline{R}_{\min}$, where $\overline{R}_{\min}$ is determined by $\overline{R}_{\min} = nw(1 - h'(nw - \overline{R}_{\min}))$.*[24]

Proof. When $w < \left(\frac{1}{1-\alpha}\right)\frac{G^*}{n}$, under any lottery mechanism with $\alpha \in [\frac{n-1}{n}, 1)$, an SPSE exists when $R \leq nw - G^V$. Of all the above lottery mechanisms with $\alpha \in [\frac{n-1}{n}, 1)$ and $R \leq nw - G^V$, Lemmas 1 and 2 state that both levels of social welfare and public good are increasing in R when wealth constraints do not bind (i.e., when $x^\alpha < w$); and both levels of social welfare and public good are strictly decreasing in R when wealth constraints bind (i.e., when $x^\alpha = w$). The above result implies that given a fixed α, the social welfare level is maximized when $R = \overline{R}(\alpha, w)$, under which $x^\alpha = w$ is just an interior solution. Moreover, it can be shown that if $\alpha \in [\frac{n-1}{n}, 1)$ is a choice variable for the designer, the maximized social welfare level—i.e., the social welfare level of the lottery mechanism with α and $R = \overline{R}(\alpha, w)$—is decreasing in α, because $\overline{R}(\alpha, w)$ increases in α, which implies that the level of public good, $nw - \overline{R}(\alpha, w)$, increases when α gets smaller. Therefore, we conclude that of all lottery mechanisms with (α, R) such that $\alpha \in [\frac{n-1}{n}, 1)$ and $R \in (0, nw - G^V]$, to maximize the social welfare, it is optimal to choose $\alpha = \frac{n-1}{n}$ and $R = \overline{R}_{\min}$. □

Theorem 2 shows that the smaller α is, the smaller $\overline{R}(\alpha, w)$ is. Intuitively, this result implies that given the total tickets sold being fixed at nw, when more can be used to finance the public good (since less is used to provide the lottery prize), the level of social welfare is higher.

In summary, Theorems 1 and 2 state the following results on the optimal nonlinear lottery mechanism: (i) When the individual wealth is sufficiently large ($w \geq G^*$), there always exists a nonlinear lottery mechanism (α, R), where $\alpha \in [\frac{n-1}{n}, 1)$ and $R = \frac{\alpha}{1-\alpha}G^*$, that induces efficient public good provision. In this case, each agent spends a proportion of her wealth on lottery tickets. (ii) When the individual wealth is sufficiently small ($w < G^*$), there exists no lottery mechanism which induces efficient public good provision; among the lottery mechanisms we study, it is optimal for the designer (who maximizes social welfare) to choose the lottery mechanism with the lowest α (i.e., $\alpha = \frac{n-1}{n}$) and, correspondingly, the lowest lottery prize R (i.e., $R = \overline{R}_{\min}$). In this case, each agent spends all her wealth on lottery tickets—as mentioned earlier, given the total wealth fixed at nw, more is used to finance the public good when less is used to provide the lottery prize R.

2.3. Asymmetric agents: The two-player case

Our analysis so far has assumed that all agents are symmetric. When agents are asymmetric, a complete characterization of a general n-player model is hard to obtain. In this subsection, we focus on a model with two asymmetric agents, and show that efficient public good provision can still be achieved by a nonlinear lottery mechanism as long as two agents are not sufficiently heterogenous.

Consider the following model with two agents, 1 and 2. Each agent has an initial wealth $w > 0$. Agent i, $\forall i \in \{1, 2\}$, has a quasilinear utility function $u_i(g_i, G) = g_i + h_i(G)$, where g_i stands for her consumption of the private good, and G stands for the level of the public good provided. Agent i experiences diminishing marginal utility from the public good—i.e., $h_i'(G) > 0$ and $h_i''(G) < 0$, for any $G \geq 0$, $\forall i \in \{1, 2\}$.

We empathize that in the current model two agents are assumed to be asymmetric in the sense that $h_1(G) \neq h_2(G)$, which is a major difference compared to the previous model with symmetric agents (in which $h_i(G) = h(G)$, $\forall i$). In particular, without loss of generality, assume that when the public good is provided efficiently, agent 1 derives a higher level of marginal utility than agent 2—i.e., $h_1'(G^*) > h_2'(G^*)$. Formally, we define

$$\rho = \frac{h_1'(G^*)}{h_2'(G^*)}, \tag{19}$$

which is the ratio of the two agents' marginal utilities under efficient public good provision. Note that $h_1'(G^*) > h_2'(G^*)$ implies that $\rho > 1$. Intuitively, ρ can be interpreted as the heterogeneity between the two agents—i.e., the greater the ρ is, the more heterogeneous/asymmetric the two agents get.

We seek to find conditions which ensure the existence of the following lottery mechanism that induces efficient public good provision in a pure strategy equilibrium. With a lottery prize $R > 0$ being exogenously given,[25] agents 1 and 2 simultaneously choose the number of tickets they want to purchase, x_1 and x_2. Agent i needs to pay x_i^α to buy x_i tickets, where

[23] This condition implies that for any $\alpha \in [\frac{n-1}{n}, 1)$, there exists no lottery mechanism (of Theorem 1) that induces efficient public good provision.

[24] It is clear to see that $\overline{R}_{\min} = \overline{R}(\alpha = \frac{n-1}{n}, w)$.

[25] In the previous model with symmetric agents, it is natural to consider an SPSE (symmetric pure strategy equilibrium) under voluntary contribution if the lottery fails. However, when agents are asymmetric, two agents have different expected payoffs in equilibrium—in which agent 1 contributes to the public good and agent 2 gets a free ride—in this case, it is more difficult to motivate agent 2 to contribute to the public good due to the high level of her outside option if the lottery fails (recall that, by contrast, in the model with symmetric agents, each agent has the same level of outside option if the lottery fails). Here, assuming an exogenously-given R (ex ante) is reasonable, since we focus on an equilibrium in which the ticket revenue collected (ex post) exactly covers R and G^*.

$\alpha < 1$, and her probability of winning R is simply $x_i/(x_1+x_2)$, given (x_1, x_2) made and fixed. In a pure strategy equilibrium we focus on, the (total) ticket revenue collected, $x_1^{*\alpha} + x_2^{*\alpha}$, is equal to the sum of (efficient) public good G^* and lottery prize R—i.e., $x_1^{*\alpha} + x_2^{*\alpha} = G^* + R$, where G^* is determined by $h_1'(G^*) + h_2'(G^*) = 1$.

The same as in the previous symmetric model, let $r = 1/\alpha$, and $e_i = x_i^{\alpha}$, which denotes the amount of the money agent i spends on lottery tickets. Thus, the number of the tickets agent i buys can be written as $x_i = e_i^r$, $\forall i \in \{1, 2\}$. We seek to find conditions that ensure the existence of a lottery mechanism (α, R) such that two agents spend e_1^* and e_2^* on lottery tickets in equilibrium, and $e_1^* + e_2^* = G^* + R$—i.e., the public good is provided efficiently.

In such an efficient lottery mechanism with (α, R), given agent 2's equilibrium contribution e_2^*, agent 1's maximization problem is:

$$\max_{e_1} u_1 = w + \frac{e_1^r}{e_1^r + e_2^{*r}} R - e_1 + h_1(e_1 + e_2^* - R). \tag{20}$$

At equilibrium, $e_1 = e_1^*$, first order condition yields that

$$\frac{e_1^{*r-1} e_2^{*r} r}{\left(e_1^{*r} + e_2^{*r}\right)^2} R - \left(1 - h_1'(e_1^* + e_2^* - R)\right) = 0. \tag{21}$$

Similarly, for agent 2, we obtain that

$$\frac{e_2^{*r-1} e_1^{*r} r}{\left(e_1^{*r} + e_2^{*r}\right)^2} R - \left(1 - h_2'(e_1^* + e_2^* - R)\right) = 0. \tag{22}$$

When public good is provided efficiently, the following two conditions hold:

$$e_1^* + e_2^* - R = G^*, \tag{23}$$

$$h_1'(G^*) + h_2'(G^*) = 1. \tag{24}$$

Using Eqs. (21)–(24), we derive that

$$\frac{e_1^{*r-1} e_2^{*r} r}{\left(e_1^{*r} + e_2^{*r}\right)^2} R = h_2'(G^*), \ \frac{e_2^{*r-1} e_1^{*r} r}{\left(e_1^{*r} + e_2^{*r}\right)^2} R = h_1'(G^*). \tag{25}$$

Both (19) and $h_1'(G^*) + h_2'(G^*) = 1$ imply that

$$h_1'(G^*) = \frac{\rho}{1+\rho}, \ h_2'(G^*) = \frac{1}{1+\rho}. \tag{26}$$

From (19) and (25), we get $e_1^* = \rho e_2^*$. Substituting (26) into (25), and using $e_1^* = \rho e_2^*$, we derive that two agents' equilibrium contribution levels—i.e., their equilibrium spending on lottery tickets—are given by:

$$e_1^* = \frac{(1+\rho)\rho^r}{(1+\rho^r)^2} rR, \ e_2^* = \frac{(1+\rho)\rho^{r-1}}{(1+\rho^r)^2} rR. \tag{27}$$

Using $r = 1/\alpha$, (23) and (27), we obtain that

$$x_1^* = \left(\frac{(1+\rho)\rho^{\frac{1}{\alpha}}}{\alpha\left(1+\rho^{\frac{1}{\alpha}}\right)^2} R\right)^{\frac{1}{\alpha}}, \ x_2^* = \left(\frac{(1+\rho)\rho^{\frac{1}{\alpha}-1}}{\alpha\left(1+\rho^{\frac{1}{\alpha}}\right)^2} R\right)^{\frac{1}{\alpha}}, \tag{28}$$

and

$$R = \frac{\alpha\left(1+\rho^{\frac{1}{\alpha}}\right)^2}{(1+\rho)^2 \rho^{\frac{1-\alpha}{\alpha}} - \alpha\left(1+\rho^{\frac{1}{\alpha}}\right)^2} G^*. \tag{29}$$

By (28), the two players' equilibrium spendings on lottery tickets are given by

$$(x_1^*)^{\alpha} = \frac{(1+\rho)\rho^{\frac{1}{\alpha}}}{\alpha\left(1+\rho^{\frac{1}{\alpha}}\right)^2} R, \ (x_2^*)^{\alpha} = \frac{(1+\rho)\rho^{\frac{1}{\alpha}-1}}{\alpha\left(1+\rho^{\frac{1}{\alpha}}\right)^2} R. \tag{30}$$

It is straightforward to see $(x_1^*)^{\alpha} > (x_2^*)^{\alpha}$ since $\rho > 1$, which implies that the minimum individual wealth that ensures the existence of such an efficient equilibrium is given by $\underline{w} = (x_1^*)^{\alpha}$. Furthermore, using $\underline{w} = (x_1^*)^{\alpha}$, (29) and (30), we obtain that

$$\underline{w} = \frac{(1+\rho)\rho^{\frac{1}{\alpha}}}{(1+\rho)^2 \rho^{\frac{1-\alpha}{\alpha}} - \alpha\left(1+\rho^{\frac{1}{\alpha}}\right)^2} G^*. \tag{31}$$

By analyzing under what conditions there exists an efficient lottery mechanism with (α, R) satisfying (29), we obtain the following results.

Proposition 3. *In the model with two asymmetric agents, if each agent's initial wealth w is sufficiently large and the two agents are sufficiently close in the sense that $w \geq \underline{w}$ and $\rho < \overline{\rho}$, where $\underline{w}$ is given by (31) and $\overline{\rho}$ (≈ 4.68) is determined by $(\overline{\rho}-1)\log\overline{\rho} - (\overline{\rho}+1) = 0$, there exists a nonlinear lottery mechanism (α, R) such that $\alpha \in [\underline{\alpha}, 1)$ where $\underline{\alpha}$ is determined by $\rho^{\frac{1}{\underline{\alpha}}} = \underline{\alpha}/(1-\underline{\alpha})$, and R is given by (29), which induces efficient public good provision in a pure strategy equilibrium.*[26]

Proof. See Appendix. □

The above proposition shows that to ensure the existence of an efficient nonlinear lottery mechanism, two sufficient and necessary conditions are needed: (i) the initial wealth w is sufficiently high in the sense that $w \geq \underline{w}$; (ii) the two agents are sufficiently close (or they are not too heterogenous) in the sense that $\rho < \overline{\rho} \approx 4.68$.

Two remarks are made regarding the above conditions, respectively. First, a similar minimum-wealth condition is also needed in the previous model with symmetric agents. Second, sufficient low heterogeneity between agents (i.e., $\rho < \overline{\rho}$) is a necessary condition for the existence of an efficient lottery mechanism—we point out that condition (ii) is reasonably moderate, because $\rho < \overline{\rho} \approx 4.68$ implies that agent1′s marginal utility (from the public good at $G = G^*$) is at most 4.68 times larger than that of agent 2.

To further illustrate the difference between efficient lottery mechanisms in the two models (with symmetric and asymmetric agents), we provide an analysis of the following two examples, in which different specific functional forms of $h_i(G)$ and (thus) different values of ρ are assumed.

Example 1. Consider the case with two symmetric agents, and $h_1(G) = h_2(G) = \frac{3}{2}\sqrt{G}$. Using (2) and (24), it can be derived that $G^* = 2.25$ and clearly $\rho = 1$. By Theorem 1, we derive that the lottery mechanism with $\alpha = 0.8$ and $R = \frac{\alpha}{1-\alpha}G^* = 9$ can induce efficient public good provision. To see this, using (15), we obtain that each agents's equilibrium spending level (on lottery tickets) is $e_1^* = e_2^* = R/(2\alpha) = 45/8 = 5.625$. It is clear to see that the equilibrium ticket revenue equals the sum of the efficient level of public good G^* and the lottery prize R, since $5.625 + 5.625 = 2.25 + 9$ implies that $e_1^* + e_2^* = G^* + R$.

Example 2. Consider the case with two asymmetric agents, and $h_1(G) = 2\sqrt{G}$, $h_2(G) = \sqrt{G}$. Using (19) and (24), it can be derived that $G^* = 2.25$ and $\rho = 2$. By Proposition 2, we can derive that the lottery mechanism with $\alpha = 0.8$ and $R \approx 13.07$ (using (29)) can induce efficient public good provision. To see this, using (27), we obtain that two agents' equilibrium spendings (on lottery tickets) are $e_1^* \approx 10.21$,$e_2^* \approx 5.11$. It is clear to see that the equilibrium ticket revenue equals the sum of the efficient level of public good G^* and the lottery prize R, since $10.21 + 5.11 = 2.25 + 13.07$ implies that $e_1^* + e_2^* = G^* + R$.

We can see that agents are assumed to be symmetric and asymmetric in Examples 1 and 2, in which $\rho = 1$ and $\rho = 2$, respectively, but with the same efficient level of the public good ($G^* = 2.25$) and the same nonlinear pricing rule ($\alpha = 0.8$) in both examples. When agents become more heterogenous from $\rho = 1$ to $\rho = 2$, under the same nonlinear rule ($\alpha = 0.8$), a larger lottery prize ($13.07 > 9$) is needed to induce the same (efficient) level of the pubic good ($G^* = 2.25$).

2.4. A more general nonlinear-pricing lottery mechanism

In the previous analysis (Sections 2.1–2.3), we focus on a lottery mechanism in which a specific nonlinear pricing rule is assumed—i.e., an arbitrary agent i needs to pay $c(x_i) = x_i^\alpha$ to buy x_i lottery tickets. In this subsection, we consider a more general nonlinear pricing rule, such that an arbitrary agent i needs to pay $c(x_i)$ to buy x_i tickets, where $c(0) = 0$, $c'(x_i) > 0$ and $c''(x_i) \leq 0$, $\forall x_i \geq 0$. In other words, player i's payment function $c(x_i)$ is increasing in x_i at a decreasing rate, with $c(0) = 0$.

We focus on a non-linear pricing lottery mechanism, denoted by $(c(\cdot), R)$, such that efficient public good provision is induced in an SPSE where each agent spends $c(x^*)$ on buying x^* tickets. In the following analysis of this section, we ignore the wealth constraint by assuming that w is sufficiently large in the sense that $w \geq c(x^*)$ always holds in equilibrium. Suppose such an equilibrium exists, given everyone else buying x^* tickets, agent i faces the following problem:

$$\max_{x_i} u_i(x_i, x^*) = w + \frac{x_i}{(n-1)x^* + x_i}R - c(x_i) + h((n-1)c(x^*) + c(x_i) - R). \tag{32}$$

It can be further derived that

$$\frac{\partial u_i}{\partial x_i} = \frac{(n-1)x^*R}{[x_i + (n-1)x^*]^2} - c'(x_i)\big(1 - h'((n-1)c(x^*) + c(x_i) - R)\big). \tag{33}$$

Efficient public good provision implies that $nc(x^*) - R = G^*$, which can be written as

$$c(x^*) = \frac{G^*}{n} + \frac{R}{n}, \tag{34}$$

[26] Note that the marginal-utility ratio ρ is defined by (19). It can be derived that $\underline{\alpha} \in (\frac{1}{2}, 1)$ for $\rho > 1$, and $\underline{\alpha}$ increases when ρ gets larger.

and $h'(G^*) = \frac{1}{n}$; in this equilibrium, it is optimal for agent i to choose the equilibrium amount. We have $\frac{\partial u_i}{\partial x_i}|_{x_i=x^*} = 0$. Using the above results and substituting $x_i = x^*$ into (33), we derive that

$$c'(x^*)x^* = \frac{R}{n}. \tag{35}$$

Combining (34) and (35), we obtain that

$$c(x^*) - c'(x^*)x^* = \frac{G^*}{n}. \tag{36}$$

Lemma 3. *For any cost function $c(\cdot)$ such that $c(0) = 0$, $c' > 0$ and $c'' \leq 0$, if there exists a lottery mechanism, denoted by $(c(\cdot), R)$, such that efficient public good provision is induced in an SPSE in which each agent purchases x^* tickets, it must be the case that x^* is uniquely determined by (36).*[27]

Proof. Define $L(x) = c(x) - c'(x)x$. Then (36) is equivalent to $L(x = x^*) = \frac{G^*}{n}$. We derive that $L'(x) = c'(x) - (c''(x)x + c'(x)) = -c''(x)x \geq 0$, since $c''(x_i) \leq 0$ by assumption, which implies that $L(x)$ increases in x, starting from $L(0) = 0$. Thus, it is safe to conclude that if there exists an x^* that makes equation $L(x) = \frac{G^*}{n}$ hold at $x = x^*$, the value of x^* must be unique. □

The above lemma shows a necessary condition for the existence of an efficient lottery mechanism under the assumption of $c' > 0$, $c'' < 0$ and $c(0) = 0$. However, this condition—i.e., the existence of x^* that solves (36)—is not sufficient to guarantee the existence of such an efficient lottery mechanism.

We provide a set of sufficient conditions in the following proposition.

Proposition 4. *Under the assumptions of $c(0) = 0$, $c' > 0$ and $c'' < 0$, there always exists a nonlinear pricing lottery mechanism denoted by $(c(\cdot), R)$, under which efficient public good provision is induced in an SPSE where each agent buys x^* tickets and $G^* = nc(x^*) - R$, provided that the following two sufficient conditions are satisfied:.*

(i) There exists an x^* such that (36) holds.[28]

(ii) Given the existence of x^*, $c'(x_i) < \frac{nx^*R}{[x_i+(n-1)x^*]^2}$ when $x_i < x^*$, and $c'(x_i) > \frac{nx^*R}{[x_i+(n-1)x^*]^2}$ when $x_i > x^*$, where $R = nc'(x^*)x^*$.

Proof. Notice that when there exists an x^* that solves (36), the value of x^* is unique by the above lemma. From (36) and $R = nc'(x^*)x^*$, we obtain that

$$\begin{aligned} nc(x^*) - R &= nc(x^*) - nc'(x^*)x^* \\ &= n(c(x^*) - c'(x^*)x^*) = n\left(\frac{G^*}{n}\right) = G^*, \end{aligned}$$

which implies that the public good is provided efficiently in equilibrium. Note that $R = nc'(x^*)x^*$ implies that $c'(x^*) = \frac{R}{nx^*}$; also note that when setting $x_i = x^*$, (33) becomes

$$\begin{aligned} \frac{\partial u_i}{\partial x_i}|_{x_i=x^*} &= \frac{(n-1)x^*R}{(nx^*)^2} - \big(1 - h'((nc(x^*) - R)\big)c'(x^*) \\ &= \frac{n-1}{n}\left(\frac{R}{nx^*} - c'(x^*)\right) = 0, \end{aligned}$$

using $h'((nc(x^*) - R) = h'(G^*) = \frac{1}{n}$ and $c'(x^*) = \frac{R}{nx^*}$. It is safe to conclude that given the existence of x^* that is determined by condition (i), we have $\frac{\partial u_i}{\partial x_i} = 0$ at $x_i = x^*$.

Next, given the existence of x^* such that $\frac{\partial u_i}{\partial x_i} = 0$ at $x_i = x^*$, we seek to show that condition (ii) ensures that $x_i = x^*$ is a global maximum, by showing that $\frac{\partial u_i}{\partial x_i} > 0$ for any $x_i < x^*$ if $c'(x_i) < \frac{nx^*R}{[x_i+(n-1)x^*]^2}$, and $\frac{\partial u_i}{\partial x_i} < 0$ for any $x_i > x^*$ if $c'(x_i) > \frac{nx^*R}{[x_i+(n-1)x^*]^2}$. By (33), we can derive that for any $x_i < x^*$,

$$\begin{aligned} \frac{\partial u_i}{\partial x_i} &= \frac{(n-1)x^*R}{[x_i + (n-1)x^*]^2} - c'(x_i)\big(1 - h'((n-1)c(x^*) + c(x_i) - R)\big) \\ &> \frac{(n-1)x^*R}{[x_i + (n-1)x^*]^2} - c'(x_i)\big(1 - h'((nc(x^*) - R)\big) \\ &= \frac{(n-1)x^*R}{[x_i + (n-1)x^*]^2} - c'(x_i)\left(1 - \frac{1}{n}\right) \\ &= \frac{n-1}{n}\left[\frac{nx^*R}{(x_i + (n-1)x^*)^2} - c'(x_i)\right] > 0 \end{aligned}$$

[27] Once x^* is obtained, R can be derived using (34).

[28] The above lemma implies the uniqueness of x^* when it exists.

if $c'(x_i) < \frac{nx^*R}{[x_i+(n-1)x^*]^2}$; and for any $x_i > x^*$,

$$\begin{aligned}\frac{\partial u_i}{\partial x_i} &= \frac{(n-1)x^*R}{[x_i+(n-1)x^*]^2} - c'(x_i)\Big(1 - h'((n-1)c(x^*) + c(x_i) - R)\Big)\\ &< \frac{(n-1)x^*R}{[x_i+(n-1)x^*]^2} - c'(x_i)\Big(1 - h'((nc(x^*) - R)\Big)\\ &= \frac{(n-1)x^*R}{[x_i+(n-1)x^*]^2} - c'(x_i)\left(1 - \frac{1}{n}\right)\\ &= \frac{n-1}{n}\left[\frac{nx^*R}{(x_i+(n-1)x^*)^2} - c'(x_i)\right] < 0\end{aligned}$$

if $c'(x_i) > \frac{nx^*R}{[x_i+(n-1)x^*]^2}$, by using the assumption of $h'' < 0$ and the fact of $h'(nc(x^*) - R) = h'(G^*) = \frac{1}{n}$. □

Intuitively, condition (i) guarantees the existence of an x^* such that agent i's marginal cost of ticket purchasing equals her marginal revenue at $x_i = x^*$. Furthermore, condition (ii) requires that an agent's marginal cost of ticket purchasing is smaller (resp. greater) than her marginal revenue before (resp. after) reaching the optimal amount x^*, which ensures that $x_i = x^*$ is a global maximum for all $x_i \geq 0$.

Actually, it is not difficult to find such an efficient nonlinear pricing rule. To see this, given that $\frac{nx^*R}{[x_i+(n-1)x^*]^2}$ decreases with x_i at a decreasing rate, condition (ii) is not too stringent in the sense that there exist a number of specific forms of cost function $c(\cdot)$ that satisfy these sufficient conditions. We provide the following example to illustrate this point.

Example 3. Consider a cost function $c(\cdot)$ that takes the following form:

$$c(x_i) = \begin{cases} -\dfrac{a}{2b}x_i^2 + ax_i & \text{if } x_i \leq \sqrt{\dfrac{2bG^*}{an}} \\ \left(-\dfrac{a}{b}\sqrt{\dfrac{2bG^*}{an}} + a\right)x_i + \dfrac{G^*}{n} & \text{if } x_i > \sqrt{\dfrac{2bG^*}{an}}, \end{cases}$$

where a and b are positive constants, which satisfy

$$b > \max\left\{\frac{2n^3}{(2n-1)^2}\left(\frac{G^*}{a}\right), \frac{(n+2)^2}{2n}\left(\frac{G^*}{a}\right)\right\}. \tag{37}$$

To facilitate the analysis, we derive the expression of the marginal cost function $c'(x_i)$ as follows:

$$c'(x_i) = \begin{cases} -\dfrac{a}{b}x_i + a & \text{if } x_i \leq \sqrt{\dfrac{2bG^*}{an}} \\ -\dfrac{a}{b}\sqrt{\dfrac{2bG^*}{an}} + a & \text{if } x_i > \sqrt{\dfrac{2bG^*}{an}}. \end{cases}$$

Note that $c'(x_i)$ decreases in x_i at a constant rate for x_i below a threshold, and stays unchanged for x_i above it. Moreover, in the appendix (Proof of Example 3), one can see that inequality (37) is a sufficient condition that ensures two functions $c'(x_i)$ and $\frac{nx^*R}{[x_i+(n-1)x^*]^2}$ cross uniquely at $x_i = x^* = \sqrt{\frac{2bG^*}{an}}$ for $x_i \in [0, \infty)$.[29] Furthermore, it can be shown that both conditions (i) and (ii) of the above proposition are satisfied in this example.

Proof. See Appendix. □

Notice that both conditions (i) and (ii) are sufficient conditions for the existence of such an efficient nonlinear lottery mechanism—however, they are not necessary conditions. To see this, for instance, the lottery mechanism proposed in Theorem 1—in which an agent i's cost of buying x_i tickets is $c(x_i) = x_i^\alpha$, where $\alpha \in [\frac{n-1}{n}, 1)$—can induce efficient public good provision in a symmetric equilibrium, but it is obvious to see that condition (ii) does not hold for sufficiently small x_i, because $c'(x_i) = \alpha x_i^{\alpha-1}$ and $\alpha < 1$ together imply that $c'(x_i) > \frac{nx^*R}{[x_i+(n-1)x^*]^2}$ when x_i is sufficiently small.

3. Conclusion

Morgan (2000) proposes a mechanism that uses lottery revenue to fund public good provision and the lottery prize, which mitigates but cannot completely eliminate free-riding problems. We propose a revised version of Morgan's lottery

[29] Intuitively, inequality (37) requires that compared to function $\frac{nx^*R}{[x_i+(n-1)x^*]^2}$, function $c'(x_i)$ decreases at a relatively slow rate in the sense that the two functions cross only once at $x_i = x^*$.

mechanism by incorporating a nonlinear pricing rule under which the price of the lottery tickets decreases when one purchases more lottery tickets. This rule leads to a decreasing marginal cost for ticket purchase and provides stronger incentives for agents to make contributions. We find that as long as the nonlinear pricing rule is designed appropriately and the agents' initial wealth level is sufficiently high, the public good can always be provided efficiently with symmetric agents. Furthermore, our analysis of a two-agent model shows that the public good can also be provided efficiently as long as the agents are not too heterogenous.

For future work, it would be interesting to test the performance of nonlinear pricing in laboratory or field experiments, as well as to extend our analyses to more complicated environments, such as a setting with incomplete information.

Declaration of Competing Interest

Disclosure Statement by Tracy Xiao Liu

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Zhewei Wang

Shandong Universit

Data availability

Data will be made available on request.

Appendix A

A1. Proof of Lemma 1

The proof of Lemma 1 consists of two steps: In step 1, we fix all others' ticket number at x^* and show that a representative agent i would also choose x^*, which dominates any x_i such that ${x_i}^\alpha + (n-1)(x^*)^\alpha \geq R$—i.e., when the lottery is held successfully. In step 2, we further show that choosing x^* dominates any x_i such that ${x_i}^\alpha + (n-1)(x^*)^\alpha < R$—i.e., when the lottery fails.

We first look at step 1. Recall that $\underline{x} = [R-(n-1)(x^*)^\alpha]^{\frac{1}{\alpha}}$ and ${x_i}^\alpha + (n-1)(x^*)^\alpha \geq R$ when $x_i > \underline{x}$. When everyone else buys lottery tickets x^*, agent i's problem is

$$\max_{x_i \in [\underline{x},\ w]} u_i(x_i, x^*) = w + \frac{x_i}{(n-1)x^* + x_i}R - x_i^\alpha + h\Big((n-1)(x^*)^\alpha + x_i^\alpha - R\Big).$$

The first-order condition is

$$\frac{(n-1)x^*}{((n-1)x^* + x_i)^2}R = \alpha x_i^{\alpha-1}\Big(1 - h'((n-1)(x^*)^\alpha + x_i^\alpha - R)\Big).$$

In equilibrium, the first-order condition holds at $x_i = x^*$, this yields that

$$\left(\frac{n-1}{\alpha n}\right)R = n(x^*)^\alpha\Big(1 - h'(n(x^*)^\alpha - R)\Big),$$

which is exactly (18). It is straightforward to show that x^* satisfying (18) must increase in R when $R \in (0, \overline{R}(\alpha, w)]$, where $\overline{R}(\alpha, w)$ is defined in (17) and $\overline{R}(\alpha, w) \in (0, nw)$. Let x^* increase and R stay unchanged, where $R \in (0, \overline{R}(\alpha, w)]$; the LHS of (18) stays unchanged and the RHS of (18) increases. Thus, we can conclude that there exists a unique x^* that satisfies (18) once $R \in (0, \overline{R}(\alpha, w)]$ is given. Following a similar procedure of showing that x^* in Theorem 1, which is given by (15), is an SPSE, it can be shown that x^* in Lemma 1, which is determined by (18), is also an SPSE.

Define $Y = n(x^*)^\alpha$, which is the level of total contribution. Then (18) can be rewritten as

$$\left(\frac{n-1}{\alpha n}\right)R = Y\left(1 - h'(Y - R)\right). \tag{A.1}$$

It is easy to obtain that with $\alpha \in [\frac{n-1}{n}, 1)$ and wealth constraint $w < G^*$ (which means that for any $\alpha \in [\frac{n-1}{n}, 1)$ we always have $w < \left(\frac{1}{1-\alpha}\right)\frac{G^*}{n}$), the efficient level of public good provision can never be achieved in equilibrium, as $Y - R < G^*$. Thus we always have $h'(Y - R) > \frac{1}{n}$, which implies that

$$\frac{n-1}{n} > 1 - h'(Y - R). \tag{A.2}$$

Next, we seek to show that when (A.1) holds, the level of the public good increases in R—i.e.,

$$\frac{d}{dR}\{Y - R\} = \frac{dY}{dR} - 1 > 0. \tag{A.3}$$

We prove the above inequality by contradiction. Assume that it is not always true—i.e., $\frac{dY}{dR} \le 1$ is true for at least some R—more specifically, assume that $\frac{dY}{dR} \le 1$ for $R \in (R_{low}, R_{high})$. Thus, for R in this interval, $Y - R$ is non-increasing in R, and $1 - h'(Y - R)$ is also non-increasing in R. Multiplying Y on both sides of (A.2) yields that

$$Y\left(\frac{n-1}{n}\right) > Y\left(1 - h'(Y - R)\right). \tag{A.4}$$

Starting from a given value of $R \in (R_{low}, R_{high})$, we consider an arbitrarily small increase of R. By checking both sides of inequality (A.4), one can see that for the same amount of increase of R, the *LHS* of (A.4) must increase more than the *RHS* does—this is because the term $1 - h'(Y - R)$ of the *RHS* is non-increasing in R, while the term $\frac{n-1}{n}$ of the *LHS* stays unchanged when R increases, with an inequality of $\frac{n-1}{n} > 1 - h'(Y - R)$, which is obtained from (A.2). Based on the above results, for any $R \in (R_{low}, R_{high})$, we can write that

$$\frac{d}{dR}\left\{Y\left(\frac{n-1}{n}\right)\right\} = \left(\frac{n-1}{n}\right)\frac{dY}{dR} \ge \frac{d}{dR}\left\{Y\left(1 - h'(Y - R)\right)\right\}. \tag{A.5}$$

Using (A.1), we also obtain that

$$\frac{d}{dR}\left\{Y\left(1 - h'(Y - R)\right)\right\} = \frac{d}{dR}\left\{\left(\frac{n-1}{\alpha n}\right)R\right\} = \frac{1}{\alpha}\left(\frac{n-1}{n}\right). \tag{A.6}$$

Using both (A.5) and (A.6), we obtain that $\left(\frac{n-1}{n}\right)\frac{dY}{dR} \ge \frac{1}{\alpha}\left(\frac{n-1}{n}\right)$, which further implies that $\frac{dY}{dR} \ge \frac{1}{\alpha} > 1$, as $\alpha < 1$. This contradicts the initial assumption of $\frac{dY}{dR} \le 1$ for $R \in (R_{low}, R_{high})$—thus, it must be the case that $\frac{dY}{dR} > 1$ for all $R \in (0, \overline{R}(\alpha, w)]$. Therefore, it is safe to conclude that the level of public good $Y - R$ increases in R for $R \in (0, \overline{R}(\alpha, w)]$.

It can be verified that when $R = 0$, the lottery mechanism turns to the case of voluntary contribution, in which $(x^*)^\alpha = \frac{1}{n}G^V > 0$. When $R = \overline{R}(\alpha, w)$, each agent spends all her wealth purchasing lottery tickets, i.e., $(x^*)^\alpha = w$, and the level of public good provision is $nw - \overline{R}(\alpha, w)$ in equilibrium. Since we have shown that both individual spending and public good provision (i.e., both $(x^*)^\alpha$ and $n(x^*)^\alpha - R$) are increasing in R, we conclude that when R increases from 0 to $\overline{R}(\alpha, w)$, $(x^*)^\alpha$ increases from $\frac{1}{n}G^V$ to w and $n(x^*)^\alpha - R$ increases from G^V to $nw - \overline{R}(\alpha, w)$.

Now we turn to Step 2, in which we seek to show that for agent i, choosing x^* dominates any x_i such that $x_i{}^\alpha + (n-1)(x^*)^\alpha < R$ and thus the lottery fails. When the lottery fails, agents go back to the case of voluntary contribution. Due to the monotonicity of $h(G) - G/n$ before G^*, we see that each agent's equilibrium payoff (under the lottery mechanism we consider) is strictly increasing in the level of the public good before reaching the efficient level G^*. It is also easy to show that the level of the public good is strictly higher when choosing x^* than choosing any $x_i < \underline{x}$ that leads to voluntary contribution, i.e., $n(x^*)^\alpha - R > G^V$, $\forall R \in (0, \overline{R}(\alpha, w)]$. This further implies that for an arbitrary agent i, choosing x^* always dominates choosing any $x_i < \underline{x}$.

A2. Proof of Lemma 2

Recall that in the proof of Lemma 1, we have shown that when $R = \overline{R}(\alpha, w)$, where $\alpha \in [\frac{n-1}{n}, 1)$, each agent spending $(x^*)^\alpha = w$ on lottery tickets constitutes an equilibrium, and given everyone else choosing $(x^*)^\alpha = w$, for an arbitrary agent i, choosing $(x_i^*)^\alpha = w$ is an interior solution for her maximization problem.

Next, we seek to show that when $R \in (\overline{R}(\alpha, w), nw - G^V]$, where $\alpha \in [\frac{n-1}{n}, 1)$, each agent spending $(x^*)^\alpha = w$ on lottery tickets also constitutes an equilibrium. It can be shown that when $R \in (\overline{R}(\alpha, w), nw - G^V]$, condition (18) never holds, so an interior optimum does not exist. It is easy to obtain that when $R \in (\overline{R}(\alpha, w), nw - G^V]$, for an arbitrary agent i, choosing $x_i = w^{\frac{1}{\alpha}}$ is optimal (as a corner solution) with the wealth constraint $(x_i)^\alpha \le w$, because at $x_i = w^{\frac{1}{\alpha}}$, her marginal revenue of increasing the number of tickets purchased is strictly larger than her marginal cost, as the LHS of (18) is strictly greater than the RHS of (18). This is an equilibrium, because when $R \in (\overline{R}(\alpha, w), nw - G^V]$, each agent has no incentive to lower the number of tickets purchased, as it would decrease her payoff, and she cannot increase the number of tickets purchased because her wealth constraint binds.

Notice that when $R = nw - G^V$, each agent's payoff in equilibrium is

$$\frac{R}{n} + h(nw - (nw - G^V)) = w - \frac{1}{n}G^V + h(G^V),$$

which exactly equals her payoff under voluntary contribution. Thus, we can conclude that a lottery mechanism with $\alpha \in [\frac{n-1}{n}, 1)$ and $\overline{R}(\alpha, w) < R < nw - G^V$ gives each agent a higher expected payoff in equilibrium than under voluntary contribution.

A3. Proof of Proposition 3

Consider a new model where everything else being the same except that $h_1(\cdot)$ and $h_2(\cdot)$ in the original model are replaced by $\widetilde{h}_1(\cdot)$ and $\widetilde{h}_2(\cdot)$, where $\widetilde{h}_i'(G) = h_i'(G^*)$ for any $G > 0$ and $\widetilde{h}_i(G^*) = h_i(G^*)$, $\forall i \in \{1, 2\}$—using (26), the expressions of $\widetilde{h}_1(\cdot)$ and $\widetilde{h}_2(\cdot)$ can be derived as follows:

$$\begin{aligned} \widetilde{h}_1(G) &= h_1(G^*) + (G - G^*)\frac{\rho}{1+\rho}, \\ \widetilde{h}_2(G) &= h_2(G^*) + (G - G^*)\frac{1}{1+\rho}. \end{aligned} \tag{A.7}$$

Due to $h_i'(\cdot) > 0$ and $h_i''(\cdot) < 0$, it is straightforward to show that $\widetilde{h}_i(G) \geq h_i(G)$ and the equality holds if and only if $G = G^*$.

By $\widetilde{h}_i(G^*) = h_i(G^*)$, it can be shown that when the public good is provided efficiently in equilibrium, the two agents' equilibrium contribution choices (e_1^*, e_2^*) in both new and original models are exactly the same, provided that (e_1^*, e_2^*) indeed constitutes an equilibrium in each model.

Following a similar procedure in Nti (1999) and Wang (2010), it can be shown that in the new model with $\widetilde{h}_i(G)$, a pure strategy equilibrium does exist for $r \in (1, \overline{r}]$, where $\overline{r}$ satisfies $\rho^{\overline{r}} = 1/(\overline{r} - 1)$, and the equilibrium choices are given by (27).[30]

In the new model with $\widetilde{h}_i(\cdot)$, given $e_2 = e_2^*$, agent 1's maximization problem is:

$$\max_{e_1} u_1 = w_1 + \frac{e_1^r}{e_1^r + e_2^{*r}}R - e_1 + \widetilde{h}_1(e_1 + e_2^* - R),$$

which, using (A.7), can be written as

$$\begin{aligned} \max_{e_1} u_1 = & \underbrace{\frac{e_1^r}{e_1^r + e_2^{*r}}R - \frac{1}{1+\rho}e_1}_{\text{varies with } e_1} \\ & + \underbrace{\left[w + h_1(G^*) + \frac{\rho(e_2^* - R - G^*)}{1+\rho}\right]}_{\text{does not vary with } e_1}. \end{aligned} \tag{A.8}$$

It can be derived that

$$\frac{\partial u_1}{\partial e_1} = \frac{e_1^{r-1}e_2^{*r}r}{\left(e_1^r + e_2^{*r}\right)^2}R - \frac{1}{1+\rho},$$

and

$$\frac{\partial^2 u_1}{\partial e_1^2} = -\frac{e_1^{-2+r}rR}{\left(e_1^r + e_2^{*r}\right)^3}[(r+1)e_1^r - (r-1)e_2^{*r}].$$

In equilibrium, $e_1 = e_1^*$ and $e_2 = e_2^*$, using $e_1^* = \rho e_2^*$, we derive that

$$\frac{\partial^2 u_1}{\partial e_1^2}|_{e_1=e_1^*, e_2=e_2^*} = -\frac{e_1^{*-2+r}e_2^{*r}rR}{(e_1^{*r} + e_2^{*r})^3}[(r+1)\rho^r - (r-1)] < 0,$$

due to $(r+1)\rho^r - (r-1) > (r+1) - (r-1) > 0$.

[30] Nti (1999) and Wang (2010) show that there always exists a pure strategy equilibrium for $r \in (0, \overline{r}]$; in this paper, we focus on the more relevant case in which $r \in (1, \overline{r}]$. We point out that $r \in (1, \overline{r}]$, where $\overline{r}$ satisfies $\rho^{\overline{r}} = 1/(\overline{r} - 1)$, implies that $\alpha \in [\underline{\alpha}, 1)$, where $\underline{\alpha}$ satisfies $\rho^{\frac{1}{\underline{\alpha}}} = \underline{\alpha}/(1 - \underline{\alpha})$.

Following a similar procedure, we derive that given $e_1 = e_1^*$, agent2′s maximization problem can be written as:

$$\max_{e_2} u_2 = \underbrace{\frac{e_2^r}{e_2^r + e_1^{*r}} R - \frac{\rho}{1+\rho} e_2}_{\text{varies with } e_2} + \underbrace{\left[w + h_2(G^*) + \frac{e_1^* - R - G^*}{1+\rho} \right]}_{\text{does not vary with } e_2}. \tag{A.9}$$

$$\frac{1 + \rho^r - r\rho^r}{(1+\rho^r)^2}$$

At equilibrium, $e_2 = e_2^*$, we can derive that

$$\frac{\partial^2 u_2}{\partial e_2^2}|_{e_1 = e_1^*, e_2 = e_2^*} = \frac{e_1^{*r} e_2^{*-2+2r} rR}{(e_1^{*r} + e_2^{*r})^3} [\rho^r (r-1)) - (r+1)] < 0,$$

because agent2′s participation constraint (i.e., choosing $e_2 = e_2^*$ weakly dominates choosing $e_2 = 0$) implies that $\rho^r(r-1) \leq 1$, which further implies that $\rho^r(r-1)) - (r+1) \leq -r < 0$.

It is clear that $e_1 = e_1^*$ is a local maximum for agent 1, given $e_2 = e_2^*$; and $e_2 = e_2^*$ is a local maximum for agent 2, given $e_1 = e_1^*$. Using (A.8) and (A.9), we can see that in terms of choosing equilibrium contribution choices, the new model is equivalent to a standard Tullock contest model with a prize R and two agents' marginal costs being $1/(1+\rho)$ and $\rho/(1+\rho)$, respectively. It can be shown that in the equivalent Tullock contest model, an agent's marginal revenue first increases and then decreases when her contribution increases from zero to infinity. Thus, for each agent, there exists at most one local maximum, which is also a global maximum provided that her participation constraint holds. Therefore, (e_1^*, e_2^*) is indeed an equilibrium in the new model because each agent's equilibrium choice is a global maximum.

In the original model with $h_i(\cdot)$, given $e_j = e_j^*$, it is clear to see that $e_i = e_i^*$ is a local maximum for agent i. For any $e_i > e_i^*$, $u_i(e_i, e_j^*) < u_i(e_i^*, e_j^*)$ because it can be show that an agent's marginal revenue decreases in e_i and the marginal cost $(1 - h_i')$ increases in e_i when $e_i > e_i^*$. When $e_i < e_i^*$, there may exist some other critical points, say $e_i = y < e_i^*$. Next, we show that it is always the case that $u_i(y, e_j^*) \leq u_i(e_i^*, e_j^*)$ if y is a critical point. We show it by contradiction: Assume that we have $u_i(y, e_j^*) > u_i(e_i^*, e_j^*)$ in the original model, then in the new model we must have $\widetilde{u}_i(y, e_j^*) > \widetilde{u}_i(e_i^*, e_j^*)$ because

$$\begin{aligned} \widetilde{u}_i(y, e_j^*) - u_i(y, e_j^*) &= \widetilde{h}_i(y, e_j^*) - h_i(y, e_j^*) \\ &> \widetilde{u}_i(e_i^*, e_j^*) - u_i(e_i^*, e_j^*) = 0 \end{aligned}$$

implies that

$$\widetilde{u}_i(y, e_j^*) - \widetilde{u}_i(e_i^*, e_j^*) > u_i(y, e_j^*) - u_i(e_i^*, e_j^*) > 0.$$

Notice that $\widetilde{u}_i(y, e_j^*) > \widetilde{u}_i(e_i^*, e_j^*)$ contradicts the fact that e_i^* is a global maximizer for agent i given $e_j = e_j^*$, thus we can safely conclude that even if there exists a critical point at $e_i = y < e_i^*$, we always have $u_i(y, e_j^*) \leq u_i(e_i^*, e_j^*)$. Therefore, (e_i^*, e_j^*) indeed constitutes an equilibrium.

In equilibrium, $e_1^* + e_2^* = R + G^*$ implies that

$$\left[\frac{(1+\rho)^2 \rho^{r-1}}{(1+\rho^r)^2} - \frac{1}{r} \right] r = \frac{G^*}{R}. \tag{A.10}$$

From the above equation, to ensure the existence of a finite $R > 0$ that induces efficient public good provision, the LHS of the above equation must be strictly positive—i.e., $\frac{(1+\rho)^2 \rho^{r-1}}{(1+\rho^r)^2} - \frac{1}{r} > 0$, which can be rewritten as

$$r > \frac{\left(\frac{1}{\rho^r} + 1\right)(\rho^r + 1)}{\left(\frac{1}{\rho} + 1\right)(\rho + 1)}.$$

By analyzing the above inequality, we can see that the *LHS* of the inequality (i.e., r) increase in r with a constant slope 1; while it can be shown that the *RHS* of the inequality also increases in r with an increasing slope. Also note that $LHS = RHS = 1$ at $r = 1$. Therefore, we can conclude that there always exists some $r \in (1, \overline{r}]$, which makes the above inequality hold if $\lim_{r \to 1} \frac{\partial RHS}{\partial r} < 1$—i.e., $\left(\frac{\rho - 1}{\rho + 1}\right) \log \rho < 1$—that is to say, the two agents are sufficiently close in the sense that $\rho \in [1, \overline{\rho})$, where $\overline{\rho}$ is determined by

$$(\overline{\rho} - 1) \log \overline{\rho} - (\overline{\rho} + 1) = 0.$$

It can be derived that $\overline{\rho} \approx 4.68$.

A4. Proof of Example 3

(i) With $x^* = \sqrt{\frac{2bG^*}{an}}$ and $c(x_i) = -\frac{a}{2b}x_i^2 + ax_i$ for $x_i \leq \sqrt{\frac{2bG^*}{an}}$, we show that (36) holds at $x_i = x^*$ since

$$c(x^*) - c'(x^*)x^* = \frac{a}{2b}x^{*2} = \frac{a}{2b}\left(\sqrt{\frac{2bG^*}{an}}\right)^2 = \frac{G^*}{n},$$

which is equivalent to saying that $c'(x_i) = \frac{nx^*R}{[x_i+(n-1)x^*]^2}$ at $x_i = x^*$.

(ii) We aim to show that $-\frac{a}{b}x_i + a < \frac{nx^*R}{[x_i+(n-1)x^*]^2}$ for any $x_i < x^*$. To facilitate our analysis, define two functions as $LHS(x_i) = -\frac{a}{b}x_i + a$ and $RHS(x_i) = \frac{nx^*R}{[x_i+(n-1)x^*]^2}$, respectively. Then, the inequality becomes $LHS(x_i) < RHS(x_i)$. Using $c(x^*) = -\frac{a}{2b}x^{*2} + ax^*$, we obtain that $R = n\left(-\frac{a}{2b}x^{*2} + ax^*\right) - G^* = nax^* - 2G^*$. Then, using $G^* = \frac{an}{2b}(x^*)^2$ which is obtained from $x^* = \sqrt{\frac{2bG^*}{an}}$, we derive that $RHS(x_i = 0) > LHS(x_i = 0)$ is equivalent to

$$\frac{nR}{(n-1)^2x^*} = \frac{n(nax^* - 2G^*)}{(n-1)^2x^*} = \frac{n^2}{(n-1)^2}(1 - \sqrt{\frac{2G^*}{abn}})a > a,$$

which holds true due to $b > \frac{2n^3}{a(2n-1)^2}G^*$ that is obtained from (37).

Also, using $LHS'(x_i) = -\frac{a}{b}$, we can derive that $RHS'(x_i) < LHS'(x_i)$ for $x_i < x^*$ requires that

$$\begin{aligned} RHS'(x_i) &= \frac{-2nx^*R}{(x_i + (n-1)x^*)^3} < \frac{-2nx^*R}{(nx^*)^3} = \frac{-2nx^*(nax^* - 2G^*)}{(nx^*)^3} \\ &= -\frac{2a}{n}\left(\sqrt{\frac{an}{2bG^*}} - \frac{1}{b}\right) < -\frac{a}{b} = LHS'(x_i), \end{aligned}$$

which holds true due to $b > \frac{(n+2)^2}{an}G^*$ that is obtained from (37).

Based on the above results—i.e., $LHS(x_i = 0) < RHS(x_i = 0)$, $LHS(x_i = x^*) = RHS(x_i = x^*)$, and $LHS'(x_i) > RHS'(x_i)$ for any $x_i < x^*$, it is safe to conclude that $LHS(x_i) < RHS(x_i)$ for any $x_i < x^*$ and $LHS(x_i) = RHS(x_i)$ if and only if $x_i = x^*$.

While for $x_i > x^*$, it is straightforward to show that $c'(x_i) = -\frac{a}{b}\sqrt{\frac{2bG^*}{an}} + a > RHS(x_i)$ for any $x_i > x^*$, since $-\frac{a}{b}\sqrt{\frac{2bG^*}{an}} + a = RHS(x^*) > RHS(x_i)$.

Lastly, notice that depending on different forms of the cost function $c(\cdot)$, there can be a probability that the lottery cannot be held when x_i is sufficiently small. In that case, the public good will be provided by the VCM. It is straightforward to show that given all other agents choosing x^*, from agent i's point of view, choosing x^* dominates any small x_i such that the lottery fails and the public good is provided by the VCM. This is because agent i's expected payoff under the VCM is strictly lower than that under the lottery mechanism where each agent chooses x^* at equilibrium and efficient public good provision is achieved.

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